# Insights into the $T_{c c}^{+}$tetraquark in a constituent quark model picture 

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## The quark model

Successful classification scheme organizing the large number of conventional hadrons


## Discoveries at $B$-factories



BABAR@SLAC (USA)


CLEO@CORNELL (USA)


BES@IHEP (China)
Explosion of related experimental activity:
Signals of exotic structures? Standard $q \bar{q}$ or $q q q$ ? Threshold cusps?

LHCb@CERN (Switzerland)

GLUEX@JLAB (USA)


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## Hidden-charm LHCb tetraquarks



## Hidden-charm LHCb tetraquarks



## The landmark of 2021: Observation of $T_{c c}^{+}$



LHCb Coll, Nature Phys. 18 (2022), 751; Nature Commun., 13 (2022) 3351.

## Analysis using a Breit-Wigner model

- Signal in $D^{0} D^{0} \pi^{+}$from primary pp-vertex.
- BW signal $\left[(D D)_{S} \pi\right.$ P-wave] +2 -body phase-space background + polynomial
- Convolution with detector resolution, rms of 400 keV .
- Model assumptions:
- $J^{P}=1^{+}$state decaying to $D D^{*}$ in S-wave
- Isoscalar $T_{c c}^{+}$due to absence of signal in $D^{0} D^{+}$and $D^{+} D^{0} \pi^{+}$.
- No isospin violation in couplings to $D^{*+} D^{0}$ and $D^{* 0} D^{+}$.
- Model results:

| Parameter | Value |
| :--- | :---: |
| $N$ | $117 \pm 16$ |
| $\delta m_{\mathrm{BW}}$ | $-273 \pm 61 \mathrm{keV} / \mathrm{c}^{2}$ |
| $\Gamma_{\mathrm{BW}}$ | $410 \pm 165 \mathrm{keV}$ |

LHCb Coll, Nature Phys. 18 (2022), 751; Nature Commun., 13 (2022) 3351.

## Analysis using an unitarized model

- Dynamic amplitude of $D D^{*} \rightarrow D D^{*}$ scattering.
- Nearly-isolated resonance below the $D^{*+} D^{0}$ threshold.
- Most precise peak position wrt threshold.
- Lifetime: $\tau \sim 10^{-20} \mathrm{~s} \hookrightarrow$ Unprecedently large for exotic hadrons.
- Model parameters:

$$
\begin{aligned}
\delta m_{\text {pole }} & =\left(-360 \pm 40_{-0}^{+4}\right) k e V \\
\Gamma_{\text {pole }} & =\left(48 \pm 2_{-14}^{+0}\right) k e V
\end{aligned}
$$

Extremely narrow state, very close to threshold $\leftrightarrow$ Strong candidate for a pure molecular state.


## In this talk...



- Analysis of the "molecular" nature of charged $T_{c c}$ state as a $D D^{*}$ system using a constituent quark model.
- Study of bottom partners $T_{b b}$.
- Reference: P. G. Ortega, J. Segovia, D. R. Entem, F. Fernández, "Nature of the doubly-charmed tetraquark $T_{c c}^{+}$in a constituent quark model", Phys. Lett. B 841 (2023), 137918. [arXiv:2211.06118 [hep-ph]].


## Constituent quark model (CQM)

- Spontaneous breaking of chiral symmetry
- Chiral invariant lagrangian

$$
\mathcal{L}=\bar{\psi}\left(i \not \partial-M\left(q^{2}\right) U^{\gamma_{5}}\right) \psi
$$

- Pseudo-Goldstone bosons

$$
\begin{aligned}
\left(\phi^{a}\right. & \left.=\left\{\vec{\pi}, K_{i}, \eta_{8}\right\}\right) \\
U^{\gamma_{5}} & =e^{i \lambda_{a} \phi^{a} \gamma_{5} / f_{\pi}} \\
& \sim 1+\frac{i}{f_{\pi}} \gamma_{5} \lambda_{a} \phi^{a}-\frac{1}{2 f_{\pi}^{2}} \phi_{a} \phi^{a}+\ldots
\end{aligned}
$$

- Constituent quark mass

$$
M\left(q^{2}\right)=m_{q} F\left(q^{2}\right)=m_{q}\left[\frac{\Lambda^{2}}{\Lambda^{2}+q^{2}}\right]
$$


C.D. Roberts, arXiv:1109.6325v1 [nucl-th]


## Constituent quark model (CQM)

Beyond the chiral symmetry breaking scale $\longleftrightarrow$ QCD perturbative effects

- Taken into account through the one-gluon-exchange (OGE) potential
- The OGE is a standard color Fermi-Breit interaction from the vertex:

$$
\mathcal{L}_{q q g}=i \sqrt{4 \pi \alpha_{s}} \bar{\psi} \gamma_{\mu} G_{a}^{\mu} \lambda^{a} \psi,
$$

- $\alpha_{s}(\mu)$ an effective scale dependent strong coupling constant

$$
\alpha_{s}(\mu)=\alpha_{0} \ln ^{-1}\left(\frac{\mu^{2}+\mu_{0}^{2}}{\Lambda_{0}^{2}}\right)
$$


J. Vijande et al. J. Phys. G31 (2005) 481.

## Constituent quark model (CQM)

Beyond the chiral symmetry breaking scale $\longmapsto$ QCD non-perturbative effects

- Linear screened confining potential

$$
V_{\mathrm{CON}}(\vec{r})=\left[-a_{c}\left(1-e^{-\mu_{c} r}\right)+\Delta\right]\left(\vec{\lambda}_{i}^{c} \cdot \vec{\lambda}_{j}^{c}\right)
$$

G.S. Bali et al. Phys. Rep. 343 (2001) 1.

G.S. Bali et al. Phys. Rev. D71 (2005) 114513.


## Constituent quark model (CQM)

Model with a large history $\rightharpoondown$ All parameters constrained from low-lying meson and baryon spectra.

- Summary of interactions for $T_{c c}^{+}$:

$$
V_{q_{i} q_{j}}=\left\{\begin{array}{l}
q q \Rightarrow V_{\mathrm{CON}}+V_{\mathrm{OGE}}+V_{\mathrm{Goldstone}} \\
Q q \Rightarrow V_{\mathrm{CON}}+V_{\mathrm{OGE}} \\
Q Q \Rightarrow V_{\mathrm{CON}}+V_{\mathrm{OGE}}
\end{array}\right.
$$

- Previous studies:
- Nucleon-Nucleon interaction:

Entem:2000mq, Valcarce:1995up, Fernandez:1993hx

- Baryon spectrum:

Valcarce:2005rr, Garcilazo:2001ck

- Meson spectrum:

Vijande:2004he, Segovia:2008zz, Segovia:2016xqb

- Meson-meson states:

Ortega:2009hj, Ortega:2020uvc, Ortega:2023pmr, Ortega:2023azl

- Baryon-meson states:

Ortega:2012cx, Ortega:2016syt, Ortega:2014eoa, Ortega:2022uyu

## Resonating Group Method (RGM)

- Interaction at quark level $\hookrightarrow$ Interaction between clusters
- 1-Hadron wave function:

$$
\phi_{A}=\phi_{A}\left(\vec{p}_{A}\right) \sigma_{A}^{S F} \xi_{A}^{c}
$$

- 2-Hadron wave function:

$$
\psi=\mathcal{A}\left[\phi_{A}\left(\vec{p}_{A}\right) \phi_{B}\left(\vec{p}_{B}\right) \chi(\vec{P}) \sigma_{A B}^{S F} \xi_{A B}^{c}\right]
$$

- Dynamics of the bound state governed by the Schrödinger equation:

$$
\begin{gathered}
\left(\mathcal{H}-E_{T}\right) \left\lvert\, \Psi>=0 \Leftrightarrow \mathcal{H}=\sum_{i=1}^{N} \frac{\vec{P}_{i}^{2}}{2 m_{i}}+\sum_{i<j} V_{i j}-T_{\mathrm{CM}}\right. \\
\left(\frac{\vec{P}^{\prime 2}}{2 \mu}-E\right) \chi\left(\vec{P}^{\prime}\right)+\int\left({ }^{\mathrm{RGM}} V_{D}\left(\vec{P}^{\prime}, \vec{P}_{i}\right)+{ }^{\mathrm{RGM}} K_{E}\left(\vec{P}^{\prime}, \vec{P}_{i}\right) \chi\left(\vec{P}_{i}\right) d \vec{P}_{i}=0\right.
\end{gathered}
$$

- Scattering state dynamics governed by Lippmann-Schwinger equation:

$$
T_{\beta}^{\beta^{\prime}}\left(z ; p^{\prime}, p\right)=V_{\beta}^{\beta^{\prime}}\left(p^{\prime}, p\right)+\sum_{\beta^{\prime \prime}} \int d p^{\prime \prime} p^{\prime \prime 2} V_{\beta^{\prime \prime}}^{\beta^{\prime}}\left(p^{\prime}, p^{\prime \prime}\right) \frac{1}{z-E_{\beta^{\prime \prime}}\left(p^{\prime \prime}\right)} T_{\beta}^{\beta^{\prime \prime}}\left(z ; p^{\prime \prime}, p\right)
$$

## RGM - Direct terms



$$
{ }^{\mathrm{RGM}} V_{D}\left(\vec{P}^{\prime}, \vec{P}\right)=\sum_{i \in A, j \in B} \int d \vec{P}_{A}^{\prime} d \vec{p}_{B}^{\prime} d \vec{\rho}_{A} d \vec{p}_{B} \phi_{A^{\prime}}^{*}\left(\vec{p}_{A}^{\prime}\right) \phi_{B^{\prime}}^{*}\left(\vec{p}_{B}^{\prime}\right) V_{i j}\left(\vec{P}^{\prime}, \vec{P}\right) \phi_{A}\left(\vec{p}_{A}\right) \phi_{B}\left(\vec{p}_{B}\right)
$$

- $V_{i j}$ the interaction at quark level given by CQM
- $i(j)$ the indices that run inside the constituentes of $A(B)$ meson.
- $\vec{p}_{A(B)}$ the relative internal momentum of the $A(B)$ meson.
- The wave functions $\phi_{A(B)}$ of the mesons act as natural cut offs for the potentials.


## RGM - Exchange terms



- Identical quarks in $T_{c c}^{+}: c \bar{q}-c \bar{q}^{\prime} \mapsto$ Exchange terms needed:

$$
\mathcal{A}=\left(1-P_{q}\right)\left(1-P_{c}\right) \multimap \Psi=\left(1-P_{q}\right)\left[\left(\phi_{A} \phi_{B}+(-1)^{L+S-s_{A}-s_{B}+I-1} \phi_{B} \phi_{A}\right) \chi\left\llcorner\sigma_{A B}^{S F} \xi_{A B}^{c}\right]\right.
$$

- ${ }^{\mathrm{RGM}} K_{E}\left(\vec{P}^{\prime}, \vec{P}\right)$ is a non-local energy-dependent exchange kernel.

$$
K_{E}\left(\vec{P}^{\prime}, \vec{P}\right)=V_{E}\left(\vec{P}^{\prime}, \vec{P}\right)-E_{T}{ }^{\mathrm{RGM}} N_{E}\left(\vec{P}^{\prime}, \vec{P}\right)
$$

- It can be separated in a potential term and a normalization term.
${ }^{R G M} V_{E}\left(\vec{P}^{\prime}, \vec{P}_{i}\right)=\int d \vec{p}_{A}^{\prime} d \vec{p}_{B}^{\prime} d \vec{p}_{A} d \vec{P}_{B} d \vec{P} \phi_{A^{\prime}}^{*}\left(\vec{p}_{A}^{\prime}\right) \phi_{B^{\prime}}^{*}\left(\vec{P}_{B}^{\prime}\right) \mathcal{H}\left(\vec{P}^{\prime}, \vec{P}\right) P_{q}\left[\phi_{A}\left(\vec{P}_{A}\right) \phi_{B}\left(\vec{p}_{B}\right) \delta^{(3)}\left(\vec{P}-\vec{P}_{i}\right)\right]$,
${ }^{\mathrm{RGM}} N_{E}\left(\vec{P}^{\prime}, \vec{P}_{i}\right)=\int d \vec{p}_{A}^{\prime} d \vec{p}_{B}^{\prime} d \vec{p}_{A} d \vec{p}_{B} d \vec{P} \phi_{A^{\prime}}^{*}\left(\vec{p}_{A}^{\prime}\right) \phi_{B^{\prime}}^{*}\left(\vec{p}_{B}^{\prime}\right) P_{q}\left[\phi_{A}\left(\vec{p}_{A}\right) \phi_{B}\left(\vec{p}_{B}\right) \delta^{(3)}\left(\vec{P}-\vec{P}_{i}\right)\right]$,
- $E_{T}$ is the total energy of the system, $\mathcal{H}$ hamiltonian from CQM.


## Details of the calculation

## Aim $\longmapsto$ Evaluate the molecular nature of the $T_{c c}^{+}$

- Coupled-channels calculation of the $J^{P}=1^{+} c c \bar{q} \bar{q}^{\prime}$ sector
- Meson-meson thresholds: $D^{0} D^{*+}(3875.10), D^{+} D^{* 0}$ (3876.51) and $D^{* 0} D^{*+}$ (4017.11).
- Meson-meson pairs can be in relative ${ }^{3} S_{1}$ and ${ }^{3} D_{1}$ partial waves.
- Energy difference between $D^{0} D^{*+}$ and $D^{+} D^{* 0}$ is $\sim 1.4 \mathrm{MeV} \hookrightarrow$ Isospin breaking effects via calculation in charged basis:

$$
\begin{aligned}
& \left|D^{* 0} D^{+}\right\rangle=-\frac{1}{\sqrt{2}}\left(\left|D^{*} D, I=1\right\rangle-\left|D^{*} D, I=0\right\rangle\right) \\
& \left|D^{*+} D^{0}\right\rangle=-\frac{1}{\sqrt{2}}\left(\left|D^{*} D, I=1\right\rangle+\left|D^{*} D, I=0\right\rangle\right)
\end{aligned}
$$

- Recalls the $X(3872)$ case studied in Ortega:2009hj:
- Same $J^{P}=1^{+}$
- Similar system $D \bar{D}^{*}(X)$ vs $D D^{*}\left(T_{c c}\right)$.
- Same direct interaction for $I=0$.
- $X$ can couple to $c \bar{c}, T_{c c}$ cannot.
- $T_{c c}$ has exchange diagrams, $X$ does not.

(a)

(b)

(c)


## Results

- We find one bound state below the lower $D^{0} D^{*+}$ threshold $\mapsto$ Binding energy of $M_{D^{0} D^{*+}}-M_{\text {pole }}=387 \mathrm{keV}$.
- Most of the attraction is due to $\pi$ and $\sigma$ exchanges, but unbound unless the exchange kernel is considered.
- The state is basically a $D^{0} D^{*+}$ molecule, with $\sim 87 \%$ probability due to its proximity to threshold. The remaining $13 \%$ corresponds to the $D^{+} D^{* 0}$ channel.
- Essentially an isoscalar ( $\sim 81 \%$ ) state $\hookrightarrow$ Sizable isospin breaking ( $\sim 19 \%$ of $I=1$ ) due to the mass difference between $D^{0} D^{*+}$ and $D^{+} D^{* 0}$ channels.
- Pole mass and partial widths:

| $E_{B}(\mathrm{keV})$ | $M-i \frac{\Gamma}{2}(\mathrm{MeV})$ | $\Gamma_{D^{0} D^{0} \pi^{+}}(\mathrm{keV})$ | $\Gamma_{D^{0} D^{+} \pi^{0}}(\mathrm{keV})$ | $\Gamma_{D^{0} D^{+} \gamma}(\mathrm{keV})$ | $\Gamma(\mathrm{keV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 387 | $3874.713-i 0$ | 49 | 26 | 6 | 84 |

- State sensitive to three-body effects $\hookrightarrow$ If a $D^{*}$ energy-dependent self-energy is taken for the $D^{*}$ meson the pole moves to 278 keV binding energy and its width drops to 42 keV .


## Additional $T_{c c}$ state in $J^{P}=1^{+}$

- Besides the $T_{c c}^{+}$below $D^{0} D^{*+}$, we also find a molecular candidate slightly below the $D^{+} D^{* 0}$ threshold in the $J^{P}=1^{+} c c \bar{q} \bar{q}^{\prime}$ sector
- Probabilities of each chanel:

| State | $\mathcal{P}_{D^{0} D^{*+}}$ | $\mathcal{P}_{D^{+D * 0}}$ | $\mathcal{P}_{D^{*+D * 0}}$ | $\mathcal{P}_{I=0}$ | $\mathcal{P}_{I=1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{c c}$ | 86.8 | 13.1 | 0.1 | 81.3 | 18.7 |
| $T_{c c}^{\prime}$ | 16.9 | 83.1 | 0.01 | 57.7 | 42.3 |

- Properties of bound states (in \%):

| State | $E_{B}$ | $M-i \frac{\Gamma}{2}$ | $\Gamma_{D^{0} D^{0} \pi^{+}}$ | $\Gamma_{D^{0} D^{+} \pi^{0}}$ | $\Gamma_{D^{0} D^{+} \gamma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{c c}$ | 387 | 3874.713 | 49 | 26 | 6 |
| $T_{c c}^{\prime}$ | 3 | $3876.507-i 0.129$ | 175 | 140 | 40 |

## Experimental line shape description



- Good description of the experimental line shape of $D^{0} D^{0} \pi^{+}$.
- Theoretical line convoluted with the detector resolution (rms of 400 keV ).
- $T_{c c}$ peak clearly visible.
- $T_{c c}^{\prime}$ peak appears as a small bump smeared by the resolution.
- The normalization via $\chi^{2}$-minimization procedure.


## Experimental line shape description (II)




- Good description of the experimental line shape of $D^{0} D^{0} \pi^{+}$.
- Theoretical line convoluted with the detector resolution (rms of 400 keV ).
- $T_{c c}$ peak clearly visible.
- $T_{c c}^{\prime}$ peak appears as a small bump smeared by the resolution.
- The normalization via $\chi^{2}$-minimization procedure.


## Scattering lengths and effective ranges

| Channel | $a_{\text {sc }}[\mathrm{fm}]$ | $r_{\text {eff }}[\mathrm{fm}]$ | $g\left[\mathrm{GeV}^{-1 / 2}\right]$ |
| :---: | :---: | :---: | :---: |
| $D^{0} D^{*+}$ | -7.14 | -0.49 | 0.12 |
| $D^{+} D^{* 0}$ | $-8.98+8.57 i$ | $0.82+0.48 i$ | 0.07 |
| $D^{* 0} D^{*+}$ | $0.20+0.02 i$ | $-6.09-6.23 i$ | $<0.01$ |

- Scattering length of the lower threshold $D^{0} D^{*+}$ fully compatible with the experimental estimation ( $a_{\mathrm{sc}}^{\mathrm{LHCb}}=-7.15(51) \mathrm{fm}$ ).
- The LHCb only gives an upper limit of $r_{\exp }>-11.9(16.9) \mathrm{fm}$ at $90(95) \% \mathrm{CL} \rightarrow$ Compatible with our $r_{\text {eff }}=-0.49 \mathrm{fm}$.
- The scattering length and effective ranges for $D^{+} D^{* 0}$ and $D^{* 0} D^{*+}$ channels are complex $\mapsto$ Indicates the existence of inelastic channels.
- The real part of the $D^{+} D^{* 0}$ scattering length is large and negative $\hookrightarrow$ Compatible with the $T_{c c}^{\prime}(3876)$ bound state.


## $T_{c c}^{+}$partners and the bottom sector (I)

| $J^{P}$ | $I$ | Mass | Width | $E_{B}$ | $\mathcal{P}_{D^{*} D^{*}}$ | Type |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{+}$ | 0 | 4018.0 | 8.15 | 0.9 | $95.6 \%$ | Resonance |
|  | 1 | 4016.9 | 0.6 | -0.2 | $98.8 \%$ | Virtual |
| $1^{+}$ | - | 4014.0 | 0 | -3.1 | $38.5 \%$ | Virtual |

- We searched for $T_{c c}^{+}$partners in alternative $J^{P}$ sectors and thresholds $\curvearrowleft$ E.g. DD in $J^{P}=0^{+}$or $D^{*} D^{*}$ in $J^{P}=0^{+}, 1^{+}$and $2^{+}$.
- No bound state was found. However, we find a virtual and resonance in $J^{P}=0^{+}$ in isospin 1 and 0 , respectively, just below the $D^{*} D^{*}$ threshold.
- Additionaly, in $J^{P}=1^{+}$, below the $D^{*} D^{*}$ threshold, a faint virtual state is found just below the $D^{*} D^{*}$ threshold.


## $T_{c c}^{+}$partners and the bottom sector (II)

- In the bottom sector we analyzed the $1^{+} \bar{b} \bar{b} q q^{\prime}$ sector.
- Coupled-channels calculation analog to that of the $T_{c c}^{+} \rightharpoondown B^{0} B^{*+}, B^{+} B^{* 0}$ and $B^{*+} B^{* 0}$ thresholds.
- Two $T_{b b}$ bound states found below the $B^{0} B^{*+}$ threshold:

| Mass | $E_{B}$ | $\mathcal{P}_{B^{0} B^{*+}}$ | $\mathcal{P}_{B^{+}+B^{* 0}}$ | $\mathcal{P}_{B^{*+}+* * 0}$ | $\mathcal{P}_{l=0}$ | $\mathcal{P}_{l=1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10582.2 | 21.9 | 47.8 | 50.0 | 2.2 | 99.99 | 0.01 |
| 10593.5 | 10.5 | 51.0 | 48.6 | 0.4 | 0.02 | 99.98 |

- We searched for further $T_{b b}$ states in $J^{P}=0^{+}$and $2^{+}$, including all meson-meson channels in a relative $S$-wave $\rightarrow B B+B^{*} B^{*}$ for $0^{+}$and $B^{*} B^{*}$ for $2^{+}$.
- We find five candidates:

| $J^{P}$ | $I$ | Mass | Width | $E_{B}$ | $\mathcal{P}_{B B}$ | $\mathcal{P}_{B^{*} B^{*}}$ | $\Gamma_{B B}$ | $\Gamma_{B^{*} B^{*}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{+}$ | 0 | 10553.0 | 0 | 6.0 | $92 \%$ | $8 \%$ | 0 | 0 |
|  | 10640.7 | 2.8 | 8.7 | $76 \%$ | $24 \%$ | 2.8 | 0 |  |
|  | 1 | 10545.9 | 0 | 13.1 | $93 \%$ | $7 \%$ | 0 | 0 |
|  | 1 | 10672.6 | 72.0 | -23.2 | $39 \%$ | $61 \%$ | 30.7 | 41.3 |
| $2^{+}$ | 1 | 10642.3 | 0 | 7.1 | - | $100 \%$ | - | 0 |

These results show a populated spectroscopy in the bottom sector, which can be detected in future searches.

## Summary

- The $T_{c c}^{+}$found as a $D^{0} D^{*+}$ molecule ( $87 \%$ ) $\mapsto E_{B}=387 \mathrm{keV} / \mathrm{c}^{2}$ and $\Gamma=81$ keV , in agreement with the experimental measurements.
- The quark content of the state forces the inclusion of exchange diagrams to treat indistinguishable quarks between the $D$ mesons, which are found to be essential to bind the molecule.
- The $D^{0} D^{0} \pi^{+}$line shape, scattering lengths and effective ranges of the molecule are also analyzed, which are found to be in agreement with the LHCb analysis.
- We search for further partners of the $T_{c c}^{+}$in other charm and bottom sectors, finding different candidates. In particular, in the charm sector we found a shallow $J^{P}=1^{+} D^{+} D^{* 0}$ molecule ( $83 \%$ ), dubbed $T_{c c}^{\prime}$, just 1.8 MeV above the $T_{c c}^{+}$state.
- In the bottom sector, an isoscalar and an isovector $J^{P}=1^{+}$bottom partners were identified as $B B^{*}$ molecules lying $21.9 \mathrm{MeV} / \mathrm{c}^{2}(I=0)$ and $10.5 \mathrm{MeV} / \mathrm{c}^{2}(I=1)$, respectively, below the $B^{0} B^{*+}$ threshold.


# Thanks for your attention. <br> Pablo García Ortega 

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- Reference:
- Nature of the doubly-charmed tetraquark $T_{c c}^{+}$in a constituent quark model, Phys. Lett. B 841 (2023), 137918. [arXiv:2211.06118 [hep-ph]].
- Recent related studies:
- Unraveling the nature of the novel $T_{c s}$ and $T_{c \bar{s}}$ tetraquark candidates - Arxiv: 2305.14430
- Exploring $T_{\psi \psi}$ tetraquark candidates in a coupled-channels formalism - Arxiv: 2307.00532


## Backslides

## $T_{c c}^{+}$partners and the bottom sector (III)



Comparison of our isoscalar $T_{b b}$ candidate with the predictions from other theoretical studies.

## Weinberg's compositeness criterion

- Following Weinberg's analysis:

$$
\begin{aligned}
& a_{\mathrm{sc}}=-\frac{2(1-Z)}{2-Z} R+\mathcal{O}\left(m_{\pi}^{-1}\right) \\
& r_{\mathrm{eff}}=-\frac{Z}{1-Z} R+\mathcal{O}\left(m_{\pi}^{-1}\right)
\end{aligned}
$$

with $R=(2 m B)^{-1}$ and $B$ the binding energy.

- Taking our values $a_{\mathrm{sc}}=-7.15 \mathrm{fm}$ and $r_{\text {eff }}=-0.49 \mathrm{fm}$ we obtain

$$
Z=1-\frac{1}{\sqrt{1+2\left|\frac{r_{\mathrm{eff}}}{a_{\mathrm{sc}}}\right|}} \sim 0.06
$$

- $Z=0.06 \longmapsto$ Mostly composite!


## Antisymmetry and OPE sign

$$
X(3872)
$$

- $J^{P C}=1^{+ \pm}$State:

$$
\left|\Psi_{D \bar{D}^{*}}^{ \pm}\right\rangle=\frac{1}{\sqrt{2}}\left(\left|D \bar{D}^{*}\right\rangle \mp\left|D^{*} \bar{D}\right\rangle\right)
$$

- Quark ordering: $c \bar{q}-q^{\prime} \bar{c}$
- Central part of OPE between $q \bar{q}$ :

$$
V_{23}(q)=V(q)\left(\vec{\sigma}_{2} \cdot \vec{\sigma}_{3}\right)\left(\vec{\tau}_{2} \cdot \vec{\tau}_{3}\right)
$$

with $\left\langle\vec{\tau}_{2} \cdot \vec{\tau}_{3}\right\rangle=2 I(I+1)-3$. Hence,

$$
\left\langle\Psi_{D \bar{D}^{*}}^{ \pm}\right| V_{23}\left|\Psi_{D \bar{D}^{*}}^{ \pm}\right\rangle \propto \pm(2 I(I+1)-3) V(Q)
$$

- Sign for $(I) J^{P C}=(0) 1^{++}, D \bar{D}^{*}+$ h.c.:

$$
\left\langle\Psi_{D \bar{D}^{*}}^{+}\right| V_{23}\left|\Psi_{D \bar{D}^{*}}^{+}\right\rangle \propto-3 V(Q)
$$

$$
T_{c c}^{+}
$$

- $J^{P C}=1^{+}$State:

$$
\left|\Psi_{D D^{*}}\right\rangle=\frac{1}{\sqrt{2}}\left(\left|D D^{*}\right\rangle+(-1)^{\prime-1}\left|D^{*} D\right\rangle\right)
$$

- Quark ordering: $c \bar{q}-c \bar{q}^{\prime}$
- Central part of OPE between $\bar{q} \bar{q}$ :

$$
V_{24}(q)=-V(q)\left(\vec{\sigma}_{2} \cdot \vec{\sigma}_{4}\right)\left(\vec{\tau}_{2} \cdot \vec{\tau}_{4}\right)
$$

with $\left\langle\vec{\tau}_{2} \cdot \vec{\tau}_{4}\right\rangle=2 I(I+1)-3$. Hence,

$$
\left\langle\Psi_{D D^{*}}\right| V_{24}\left|\Psi_{D D^{*}}\right\rangle \propto(-1)^{\prime}(2 I(I+1)-3) V(Q)
$$

- Sign for $(I) J^{P}=(0) 1^{+}, D D^{*}$ :

$$
\left\langle\Psi_{D D^{*}}\right| V_{24}\left|\Psi_{D D^{*}}\right\rangle \propto-3 V(Q)
$$ with $Q$ the transferred momentum between mesons and $V(q)=\frac{1}{(2 \pi)^{3}} \frac{g_{c h}^{2}}{4 m_{q}^{2}} \frac{1}{3} \frac{\Lambda^{2}}{\Lambda^{2}+q^{2}} \frac{q^{2}}{q^{2}+m^{2}}$.

## Calculation of partial decay widths

- The $T_{c c}^{+}$only decays strongly if the $D^{*}$ inside the $D D^{*}$ disintegrates.
- As the $D^{*}$ width is small, the decay can be calculated perturbatively considering the $D^{*}$ as unstable into $D \pi$ or $D \gamma$. E.g.:

$$
\Gamma_{D^{0} D^{0} \pi^{+}}=\Gamma_{D^{*+} \rightarrow D^{0} \pi^{+}} \int_{0}^{k_{\max }} k^{2} d k\left|\chi_{D^{0} D^{*+}}(k)\right|^{2} \frac{\left(M_{T}-E_{D^{0}}-E_{D^{*+}}\right)^{2}}{\left(M_{T}-E_{D^{0}}-E_{D^{*+}}\right)^{2}+\frac{\Gamma_{D^{*}}^{2}}{4}},
$$

where

- $\Gamma_{D^{*+} \rightarrow D^{0} \pi^{+}}$is the $D^{*+}$ experimental partial width to $D^{0} \pi^{+}$.
- $\chi_{D^{0} D^{*+}}(k)$ is the wave function of the channel $D^{0} D^{*+}$
- $E_{D}$ are the total energies of the mesons involved in the reaction.
- $k_{\max }$ is the maximum on-shell momentum of the $D^{0} D^{0} \pi^{+}$system:

$$
k_{\max }=\frac{1}{2 M_{T}} \sqrt{\left[M_{T}^{2}-\left(2 m_{D^{0}}+m_{\pi^{+}}\right)^{2}\right]\left[M_{T}^{2}-m_{\pi^{+}}^{2}\right]}
$$

where $M_{T}$ is the mass of the $T_{c c}^{+}$.

- The $D^{0} D^{0} \pi^{+}$threshold is located at about 3869 MeV , i.e. there is not much phase space available, which explains the small partial width obtained.


## Scattering lengths and effective ranges

Table: $D^{0} D^{*+}$ pole position and S-wave scattering lengths for coupled-channels calculation with an unstable $D^{*}$ meson, considering different $D^{* 0}$ and $D^{*+}$ widths. We distinguish between the scattering length evaluated at the real $M_{D^{0} D^{*}+}=m_{D^{0}}+m_{D^{*}+}$ (third column) or at the complex $E_{D^{0} D^{*}+}=m_{D^{0}}+m_{D^{*}+}-i \Gamma_{D^{*}+} / 2$ (forth column).

| Case | $E_{R}-i \Gamma_{R} / 2$ | $-\pi \mu T\left(M_{D^{0} D^{*}+}\right)$ | $a_{\mathrm{sc}, \mathrm{D}^{0} D^{*+}}$ |
| :--- | :---: | :---: | :---: |
| $\Gamma_{D^{* 0}}=\Gamma_{D^{*+}}=0$ | $3874.713-i 0$ | $-7.14+0.00 i$ | $-7.14+0.00 i$ |
| $\Gamma_{D^{* 0}}=0, \Gamma_{D^{*+}}=83.4 \mathrm{keV}$ | $3874.713-i 0.036$ | $-8.64+2.32 i$ | $-7.14-0.08 i$ |
| $\Gamma_{D^{* 0}}=\Gamma_{D^{*}+}=83.4 \mathrm{keV}$ | $3874.713-i 0.042$ | $-8.58+2.43 i$ | $-7.14+0.001 i$ |

## Experimental line shape of $D^{+} D^{0} \pi^{+}$



- Good description of the experimental line shape of $D^{+} D^{0} \pi^{+}$.
- $T_{c c}^{\prime}$ not visible $\mapsto$ No bound state in $D^{+} D^{*+}$ system.

