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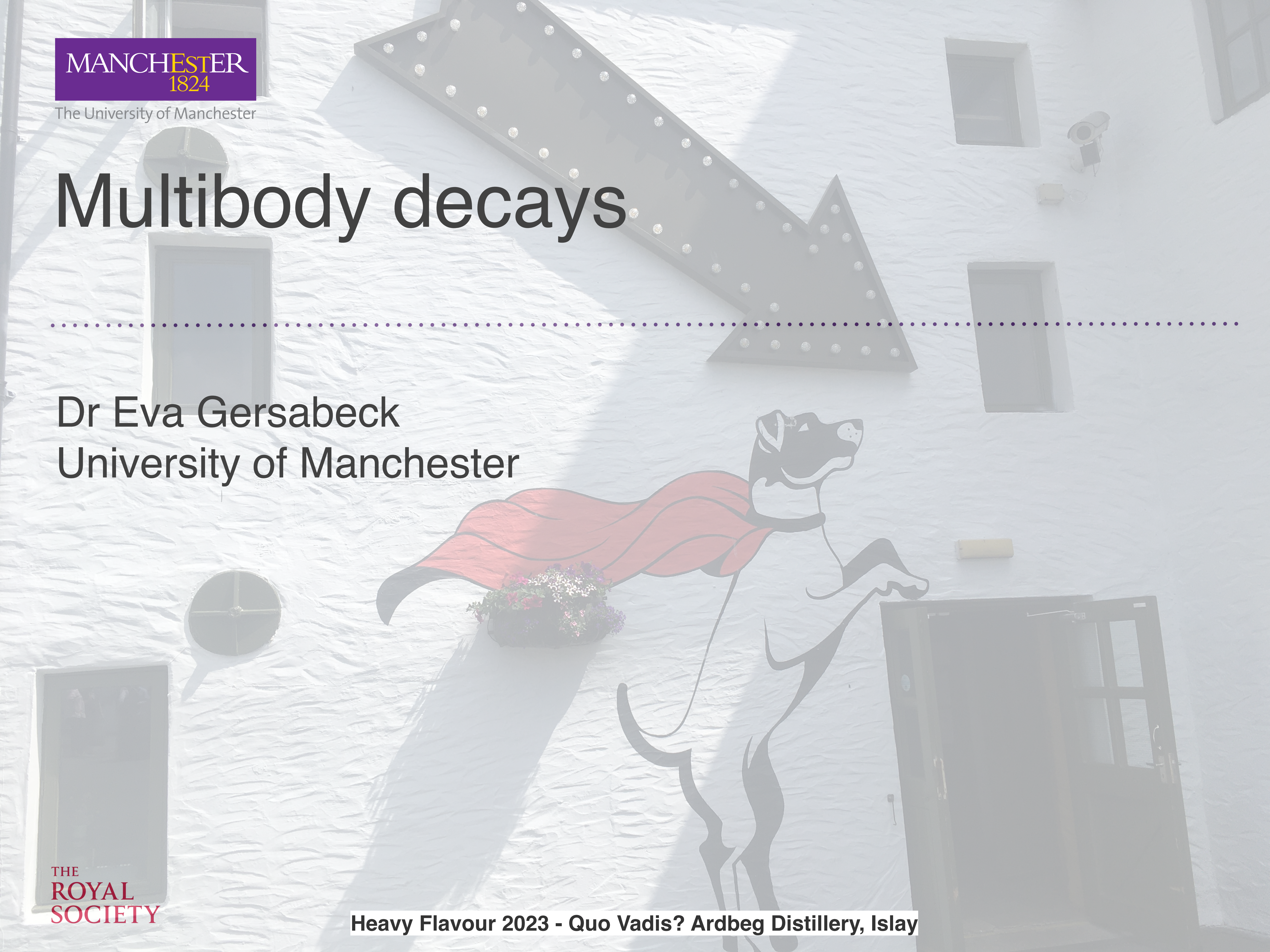

Multibody decays

Dr Eva Gersabeck
University of Manchester

THE
ROYAL
SOCIETY

Heavy Flavour 2023 - Quo Vadis? Ardbeg Distillery, Islay

Outline

- Model independent methods
    - The Energy test
    - Earth movers distance
    - T-odd moments
    - binned methods (Miranda method and co.)
    - Phase space integrated results
- Model dependent methods
- Things I can not cover
    - Input from multibody decays to γ

Covered in the talks of
Tim, Jonas and
Wolfgang

Introduction

- Multibody decays: final states are reached mainly through resonances

- Unique sensitivity to phases

- Excellent environment for CP violation and mixing: strong-phase differences varying across the Dalitz plot enhance the sensitivity

$$\mathcal{A}_{CP}(P \to f) \equiv \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} \propto \sum_{i,j} |A_i||A_j| \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j)$$

- Tests of QCD (spectroscopy, amplitude models)

- Huge samples: a bless and a curse

MODEL INDEPENDENT METHODS

Why go model independent?

- Fast discovery tools
- Binned or unbinned methods
- Can be used for direct and indirect CP violation tests
    - Will cover direct CP violation today
    - By design sensitive to local asymmetries rather than to global asymmetries

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The Energy test

Energy test

- The Energy test uses a distance function ψ_{ij} to compute a T value
- T compares the average distance between pairs of events in the phase space

$$T = \sum_{i,j>1}^{n} \frac{\psi_{ij}}{n(n-1)} + \sum_{i,j>1}^{\bar{n}} \frac{\psi_{ij}}{\bar{n}(\bar{n}-1)} - \sum_{i,j}^{n,\bar{n}} \frac{\psi_{ij}}{n\bar{n}}$$

Average distance in the first sample

Average distance in the second sample

Average distance between the two samples

- The distance function

$$\psi(d_{ij}) = e^{-d_{ij}^2/2\delta}$$

Phase space distance

$$d_{ij}^2 = \sum_{k=1}^{D} (x_{k,i} - x_{k,j})^2$$

Our case

$$x_{k,i} = m_{k,i}^2$$

Eva Gersabeck

Optimising the sensitivity

- δ is a tunable distance parameter describing the effective phase-space radius where a local asymmetry is measured

- δ is analogous to the bin size in a binned approach

- It must be:
  - Larger than the resolution of d_{ij}
  - Small enough not to dilute local asymmetries
  - Optimised value from sensitivity studies

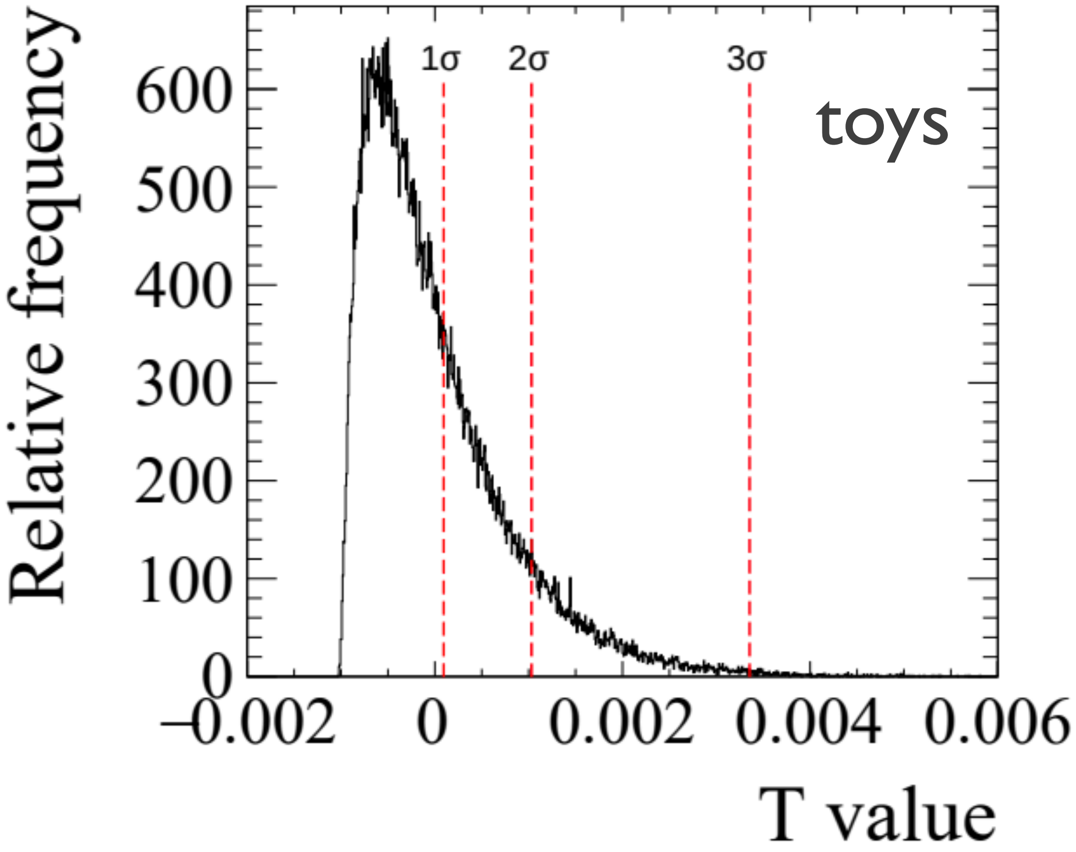
The Energy test in a nutshell

Used in: Phys. Rev. D102 (2020) 051101

Phys. Lett. B740 (2015) 158

Phys. Lett. B769 (2017) 345

- Split sample is D^0 and $\bar{D}^0$ decays
- Compute reference T value
- Compute T values from permuted samples using random flavour tags (null hypothesis)
- Compute P-value = fraction of permuted T values > reference T value



Sensitivity studies

- To verify that and how much the Energy test is sensitive to CP violation:
  - Simulate samples with comparable size to the Run 2 data samples
  - Input different amplitude and phase asymmetries in different resonances (e.g. 1%, 2%, 5%, 10% or 1°,2°,5°, ….)
  - Run the Energy test
  - Reset and repeat for a a set of δ values (i.e. perform a so called “δ-scan”)
  - Plot the P-value distributions
  - Choose the δ value (or values) that ensures the lowest P-values (i.e. the best sensitivity)

Sensitivity studies

[arXiv:2306.12746]

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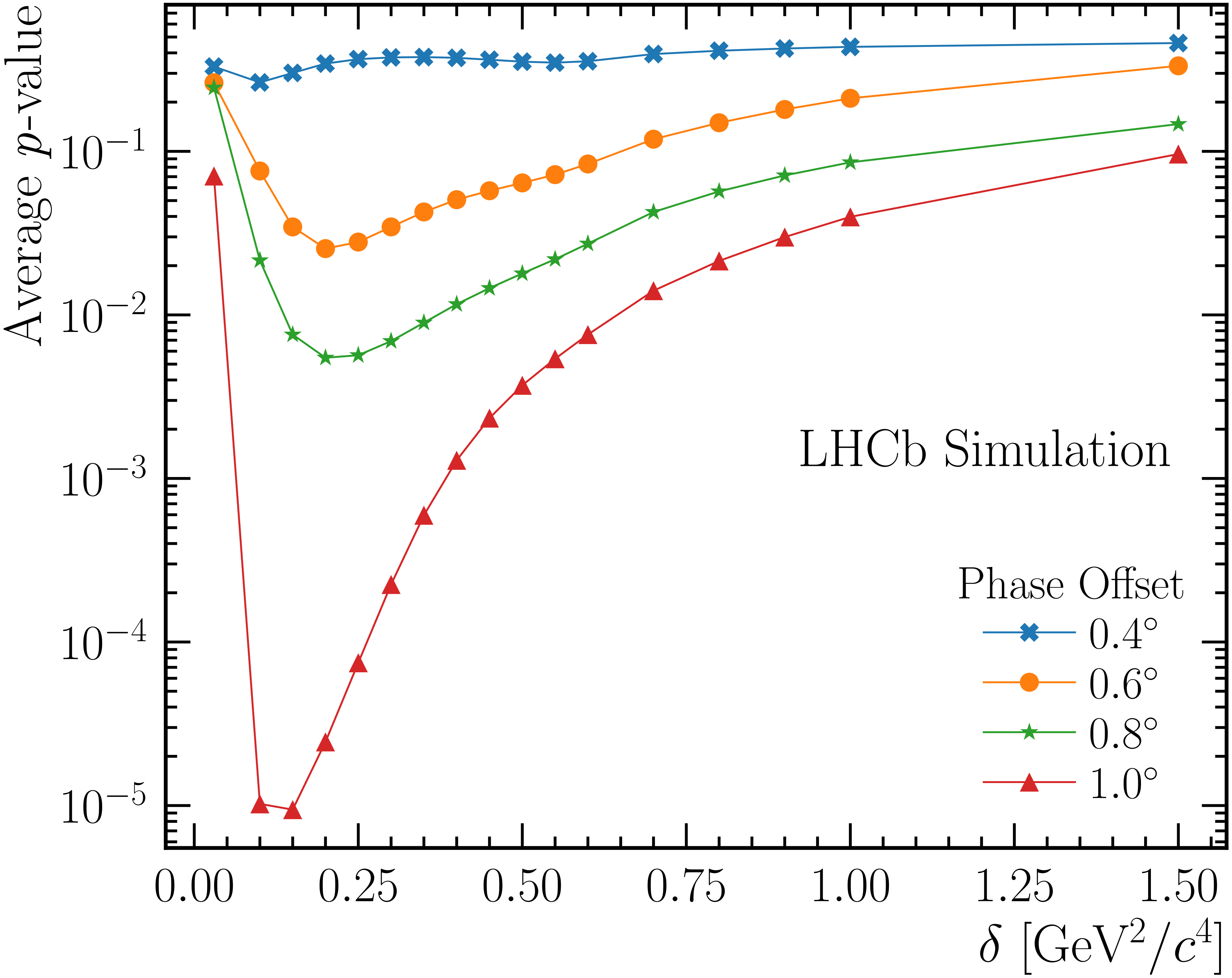


Validation of the Energy test

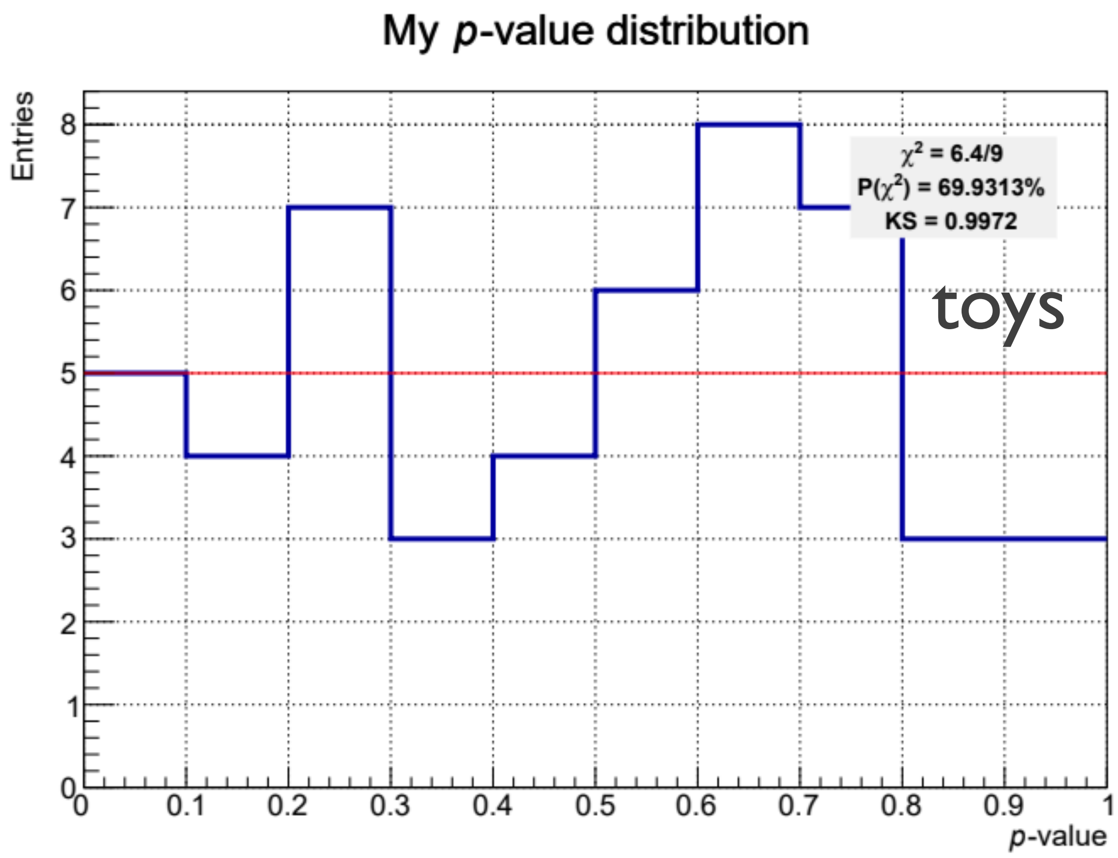
- To validate the Energy test is insensitive to instrumentation asymmetries a control channel is needed:
  - Same/ similar final state particles
  - No CP violation expected
  - High statistics

CF decays are great control samples

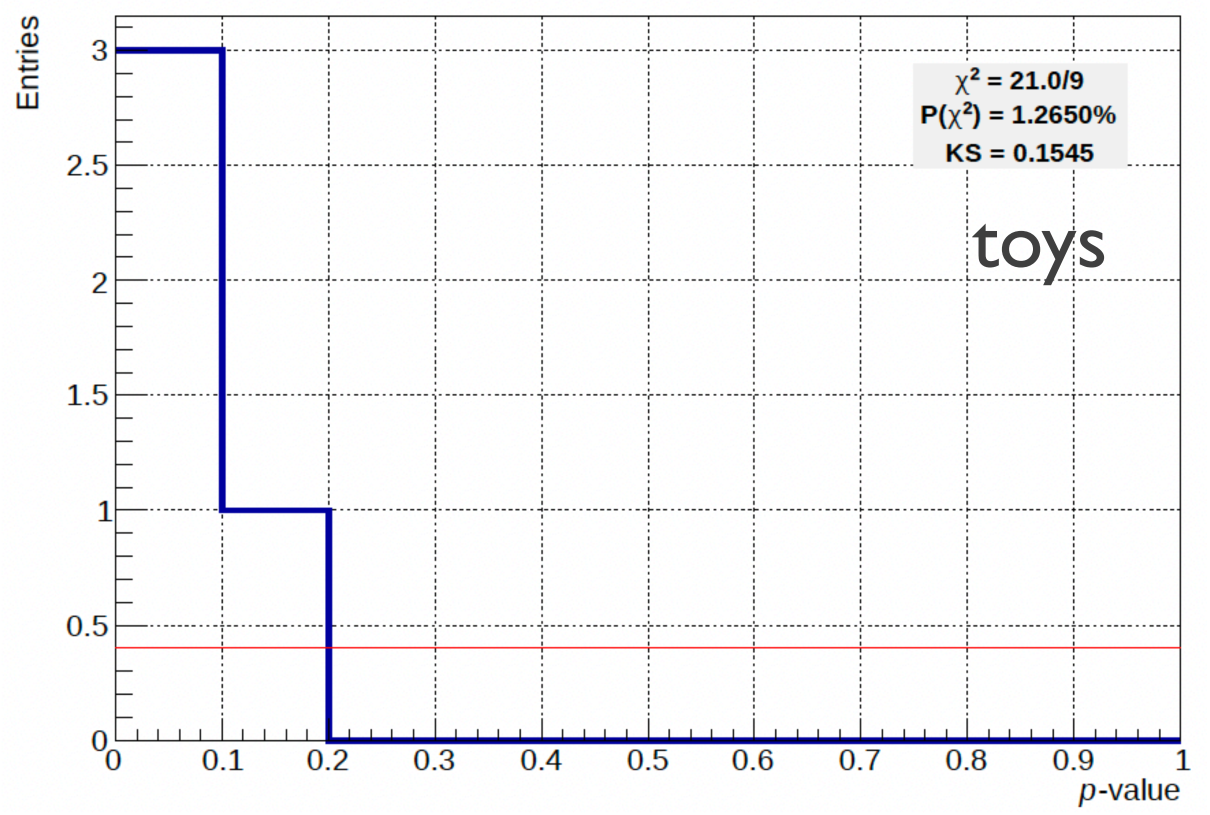
- Apply signal requirement to control channels
  - Split into n subsamples with signal sample statistics
  - Run Energy test with optimised δ value
  - Compute and plot the P-values

Symmetric or not?

- For visualisation only (toys):



Symmetric: flat distribution of the p-values



Asymmetric: p-values accumulate in the first bin

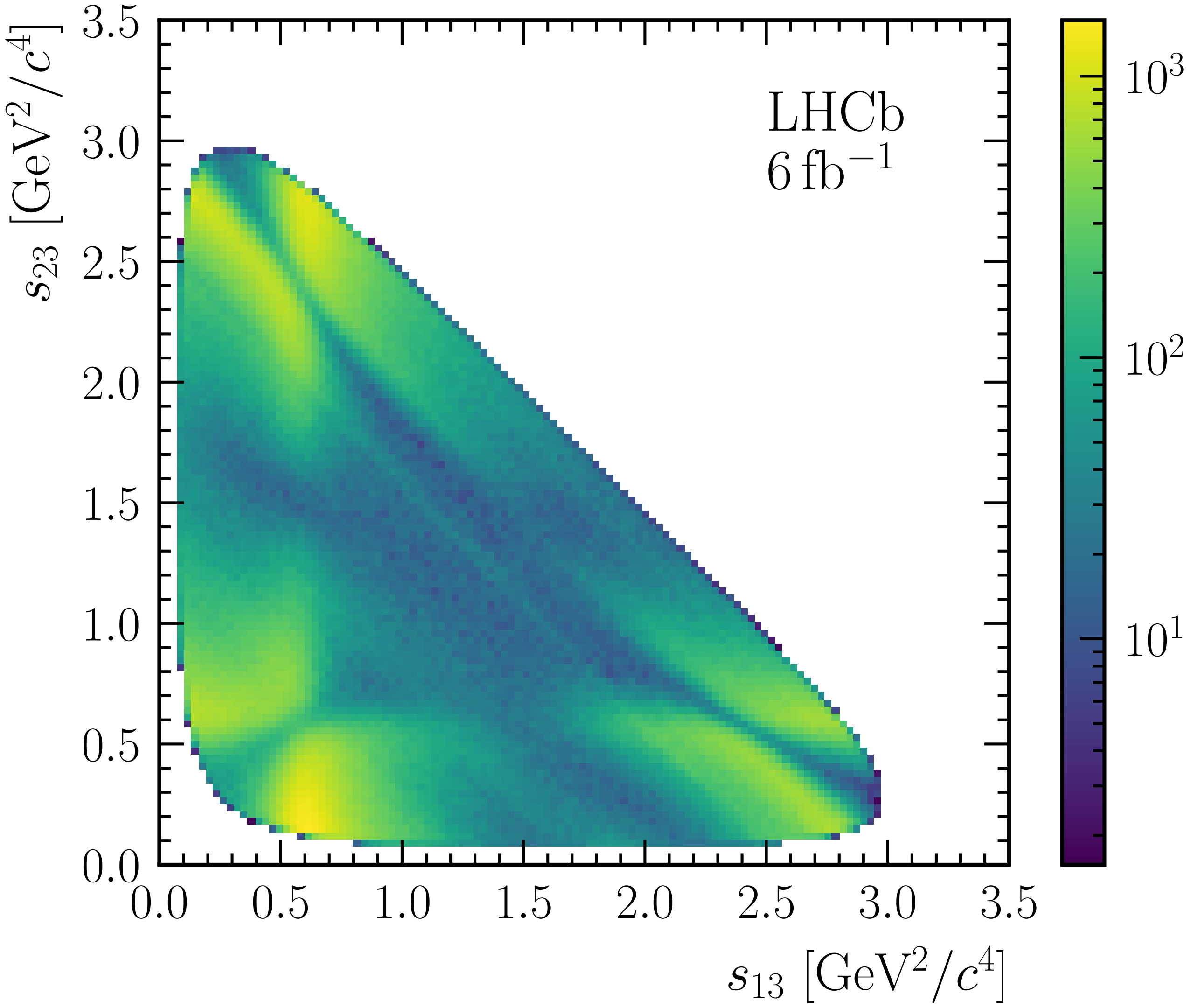
Search for CP violation in $D^0 \rightarrow \pi^+\pi^-\pi^0$

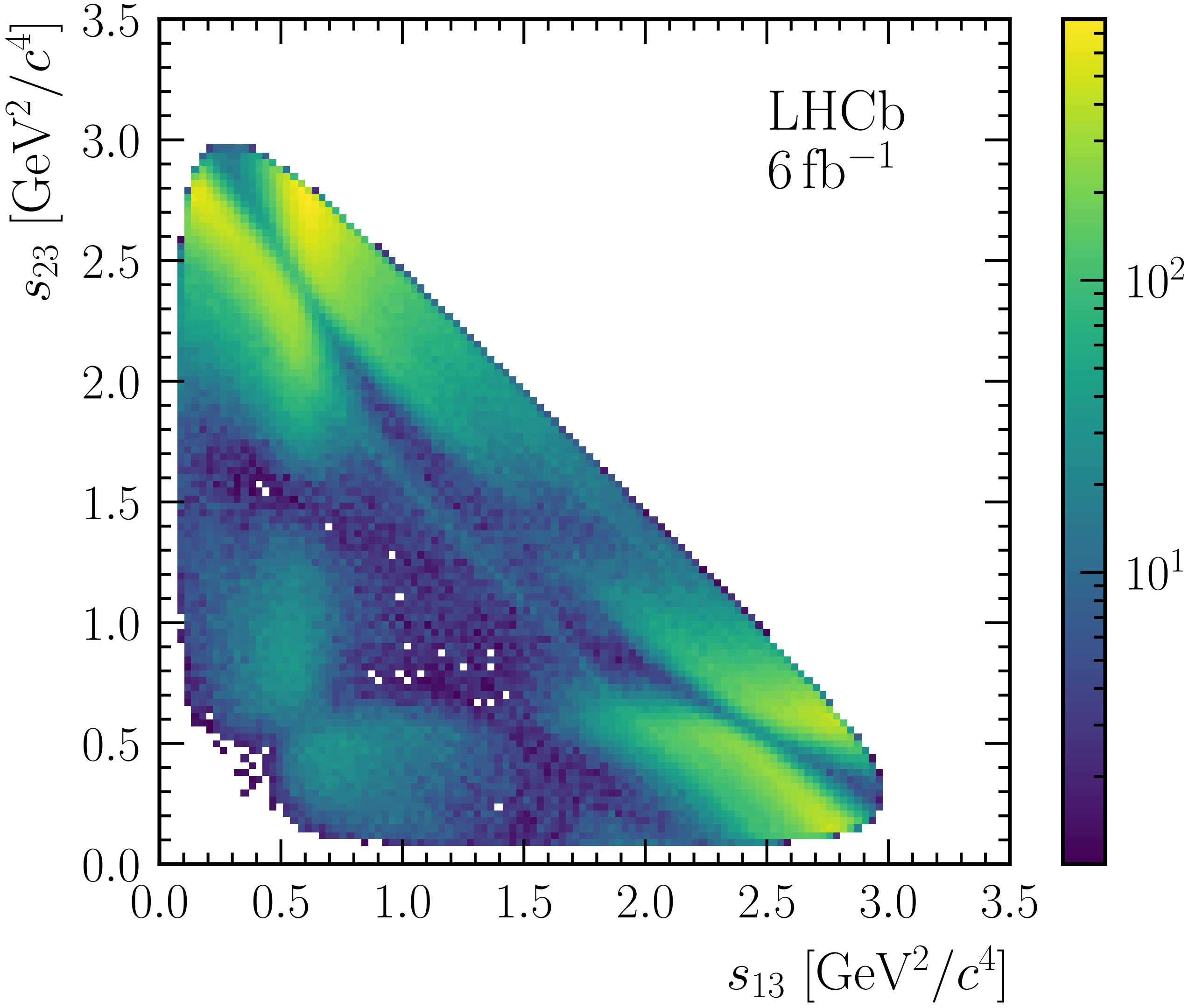
[arXiv:2306.12746]

- Singly Cabibbo suppressed D^0 decays
- Prompt sample tagged by $D^{*+} \rightarrow D^0\pi^+$
- Signal purity 81% for resolved π^0 and 91% for merged π^0
- LHCb Run 2 data (6 fb^{-1})
- Four times larger than the Run 1 sample (PLB 2014 11 043)
- Control sample: $D^0 \rightarrow K^-\pi^+\pi^0$
- Cross checks

Dalitz plot distributions

[arXiv:2306.12746]





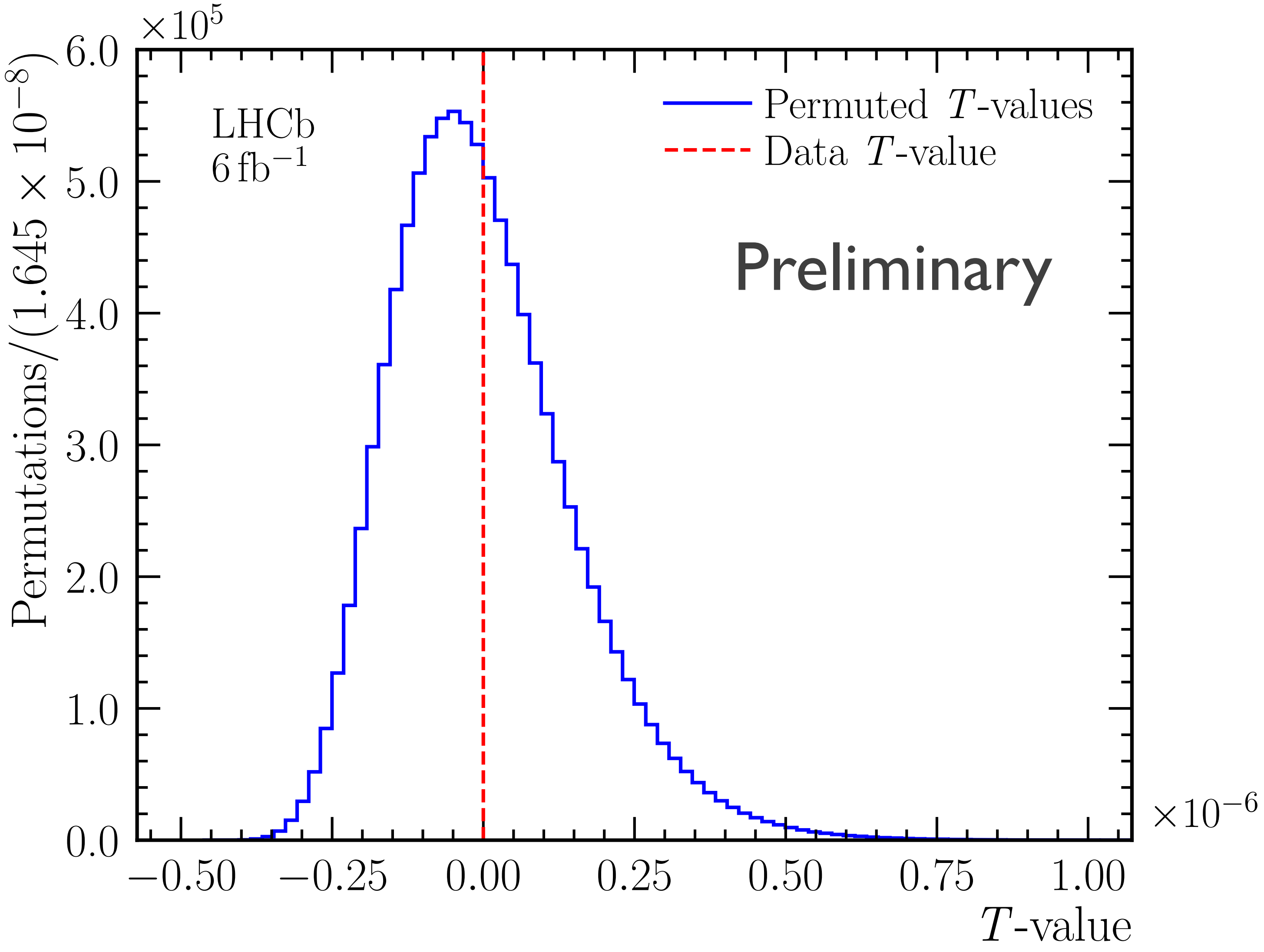
Resolved π^0
1.7 M

Merged π^0
0.8 M

CP violation results in $D^0 \rightarrow \pi^+\pi^-\pi^0$

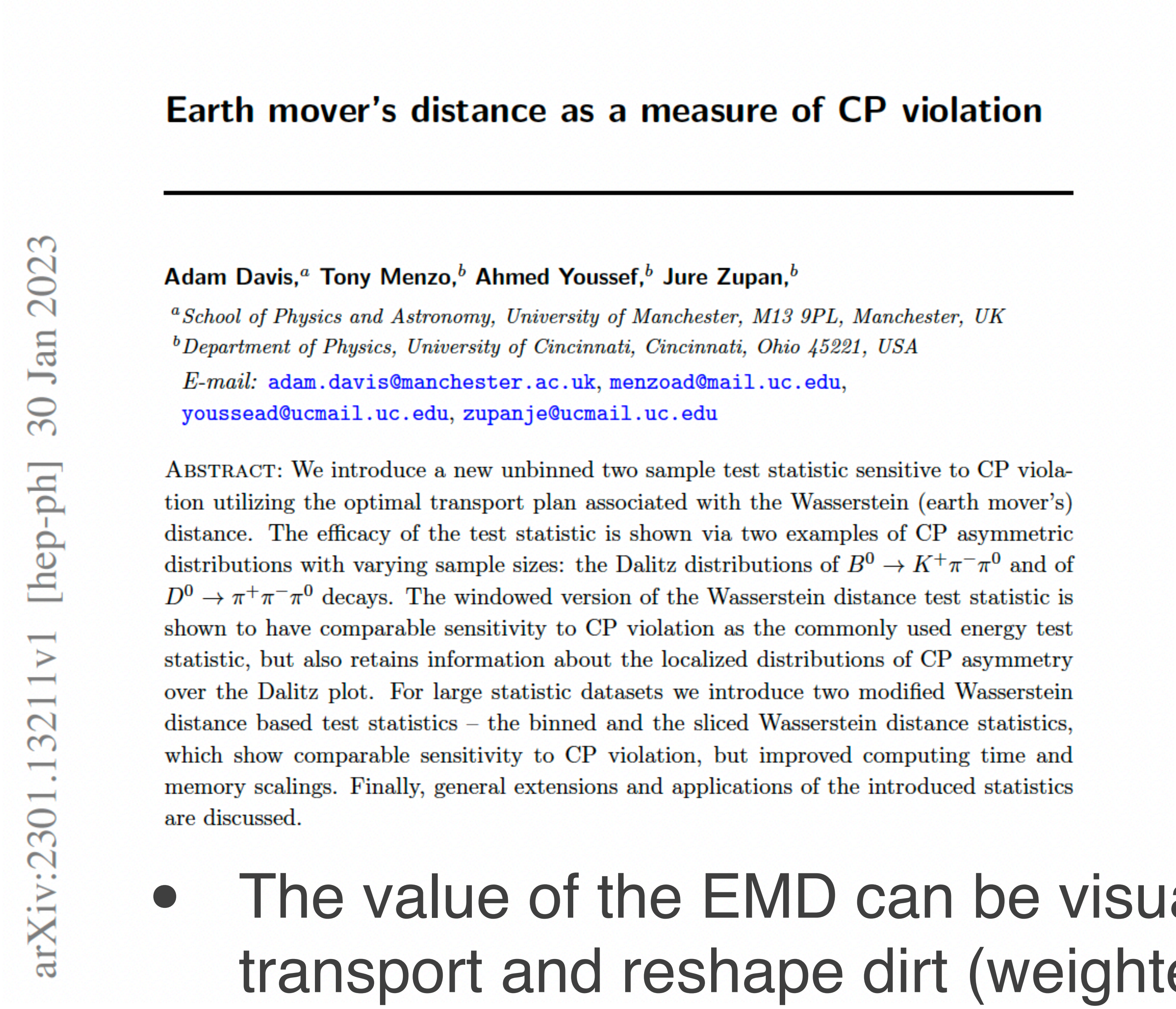
[arXiv:2306.12746]

- No evidence for local CP violation
- p-value = 61%



Earth movers distance

New model independent method



Earth mover's distance as a measure of CP violation

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[b] *Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221, USA*

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ABSTRACT: We introduce a new unbinned two sample test statistic sensitive to CP violation utilizing the optimal transport plan associated with the Wasserstein (earth mover's) distance. The efficacy of the test statistic is shown via two examples of CP asymmetric distributions with varying sample sizes: the Dalitz distributions of $B^0 \to K^+\pi^-\pi^0$ and of $D^0 \to \pi^+\pi^-\pi^0$ decays. The windowed version of the Wasserstein distance test statistic is shown to have comparable sensitivity to CP violation as the commonly used energy test statistic, but also retains information about the localized distributions of CP asymmetry over the Dalitz plot. For large statistic datasets we introduce two modified Wasserstein distance based test statistics – the binned and the sliced Wasserstein distance statistics, which show comparable sensitivity to CP violation, but improved computing time and memory scalings. Finally, general extensions and applications of the introduced statistics are discussed.

distance metric for EMD: q=1

$$W_q(\mathscr{E}, \bar{\mathscr{E}}) = \left[min \sum_{i=1}^{N} \sum_{j=1}^{\bar{N}} f_{ij} (d_{ij})^q \right]^{1/q}$$

$$\sum_{i}^{N} f_{ij} = \frac{1}{\bar{N}}, \quad \sum_{i}^{\bar{N}} f_{ij} = \frac{1}{N}, \quad \sum_{i,j}^{N,\bar{N}} f_{ij} = 1$$

- The value of the EMD can be visualised as the work required to transport and reshape dirt (weighted samples) in the form of one distribution into the form of a second distribution.
- Toys for $B^0 \to K^+\pi^-\pi^0$ and $D^0 \to \pi^+\pi^-\pi^0$ generated with Laura++, no experimental results yet but looking forward

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T-odd moments

T-odd moments method

Using triple product of final state particle momenta

$$C_T \equiv \vec{p}_1(\vec{p}_2 \times \vec{p}_3)$$

$\bar{C}_T$: the triple product for the charge conjugate state

Define triple product asymmetries

$$A_T \equiv \frac{\Gamma_{D^0}(C_T > 0) - \Gamma_{D^0}(C_T < 0)}{\Gamma_{D^0}(C_T > 0) + \Gamma_{D^0}(C_T < 0)}, \qquad \overline{A}_T \equiv \frac{\Gamma_{\overline{D}^0}(-\overline{C}_T > 0) - \Gamma_{\overline{D}^0}(-\overline{C}_T < 0)}{\Gamma_{\overline{D}^0}(-\overline{C}_T > 0) + \Gamma_{\overline{D}^0}(-\overline{C}_T < 0)},$$

$$a_{CP}^{T-odd} \equiv \frac{1}{2}(A_T - \bar{A}_T) \qquad a_P^{T-odd} \equiv \frac{1}{2}(A_T + \bar{A}_T)$$

Triple product asymmetries ∼sinϕ cosδ

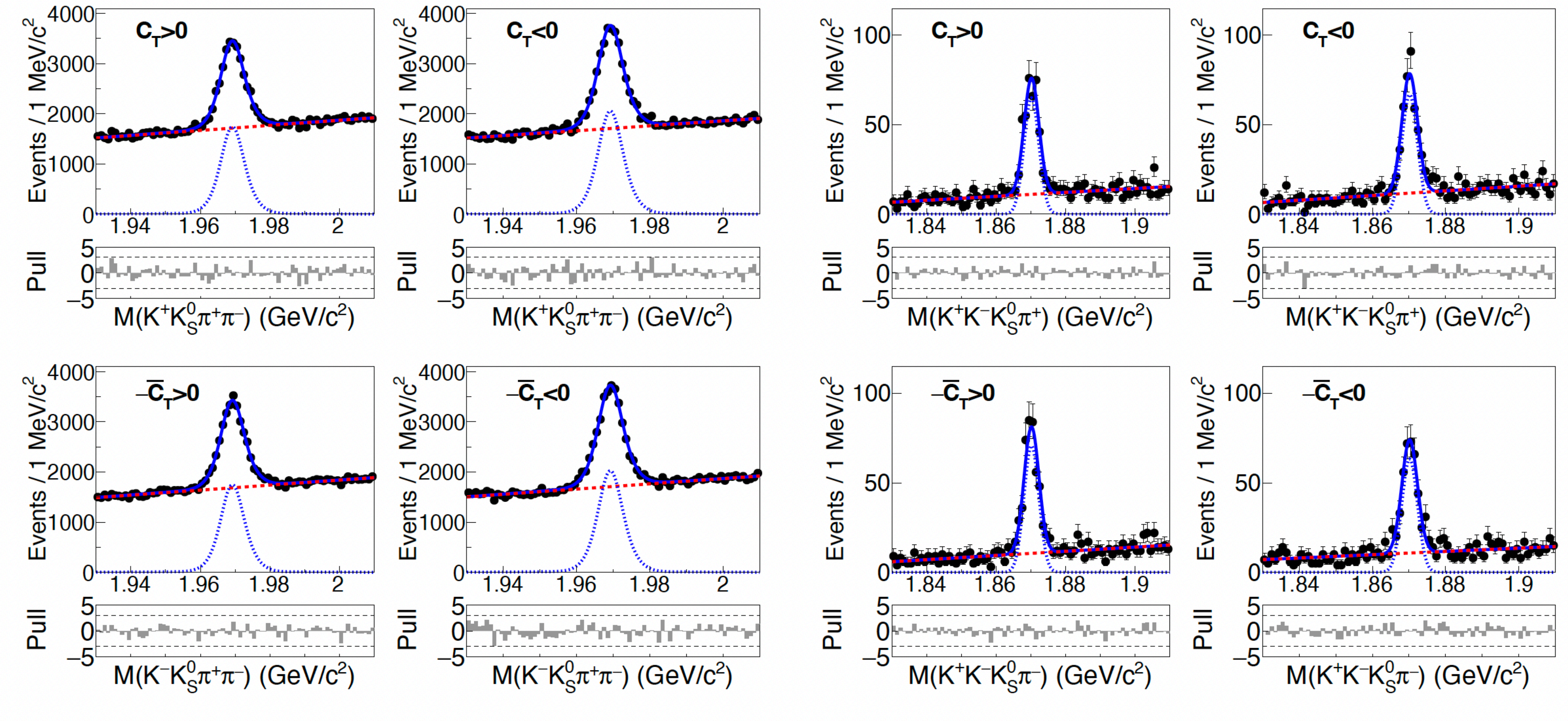
More careful consideration given in Durieux, Grossman
Phys. Rev. D 92, 076013 (2015)

All production and detection effects cancel
T: reverses the momentum, similar to P
(no real time reversal)

Used in: BaBar: PRD 81 (2010) 111103
BaBar PRD 84, 031103 (R) (2011)
FOCUS: PLB 622 (2005) 239-248
LHCb: JHEP 1410 (2014) 005
LHCb: Phys. Rev. D102 (2020) 051101

T-odd moments in $D^+_{(s)} \to K^+K^0_S h^+h^-$

arXiv:2305.11405v1

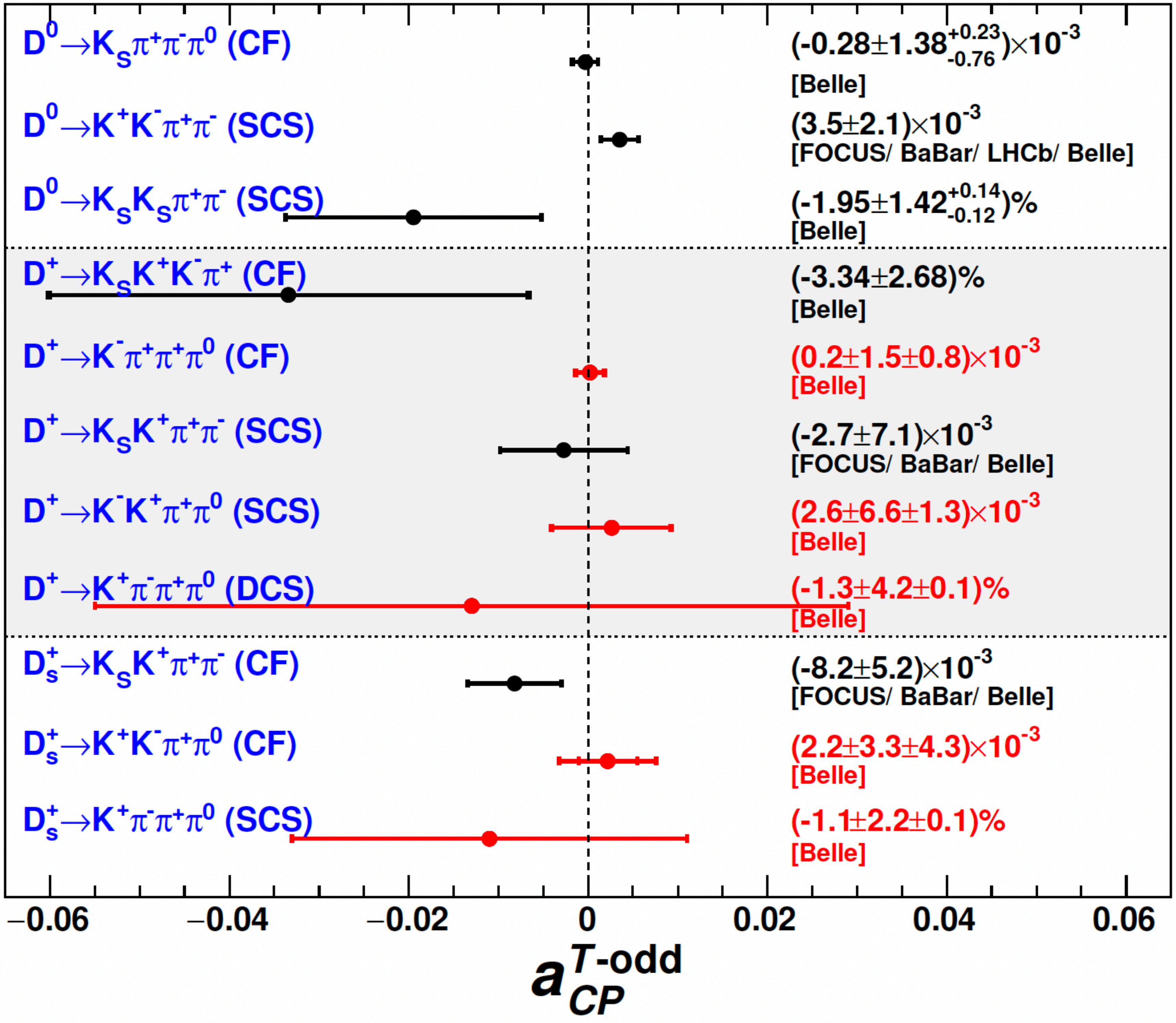


$$a_{CP}^{T\text{-odd}}(D^+ \to K^+K^0_S\pi^+\pi^-) = \quad (0.34 \pm 0.87 \pm 0.32)\%$$
$$a_{CP}^{T\text{-odd}}(D^+_s \to K^+K^0_S\pi^+\pi^-) = (-0.46 \pm 0.63 \pm 0.38)\%$$
$$a_{CP}^{T\text{-odd}}(D^+ \to K^+K^-K^0_S\pi^+) = (-3.34 \pm 2.66 \pm 0.35)\%,$$

T-odd moments in other $D^+_{(s)}$ modes

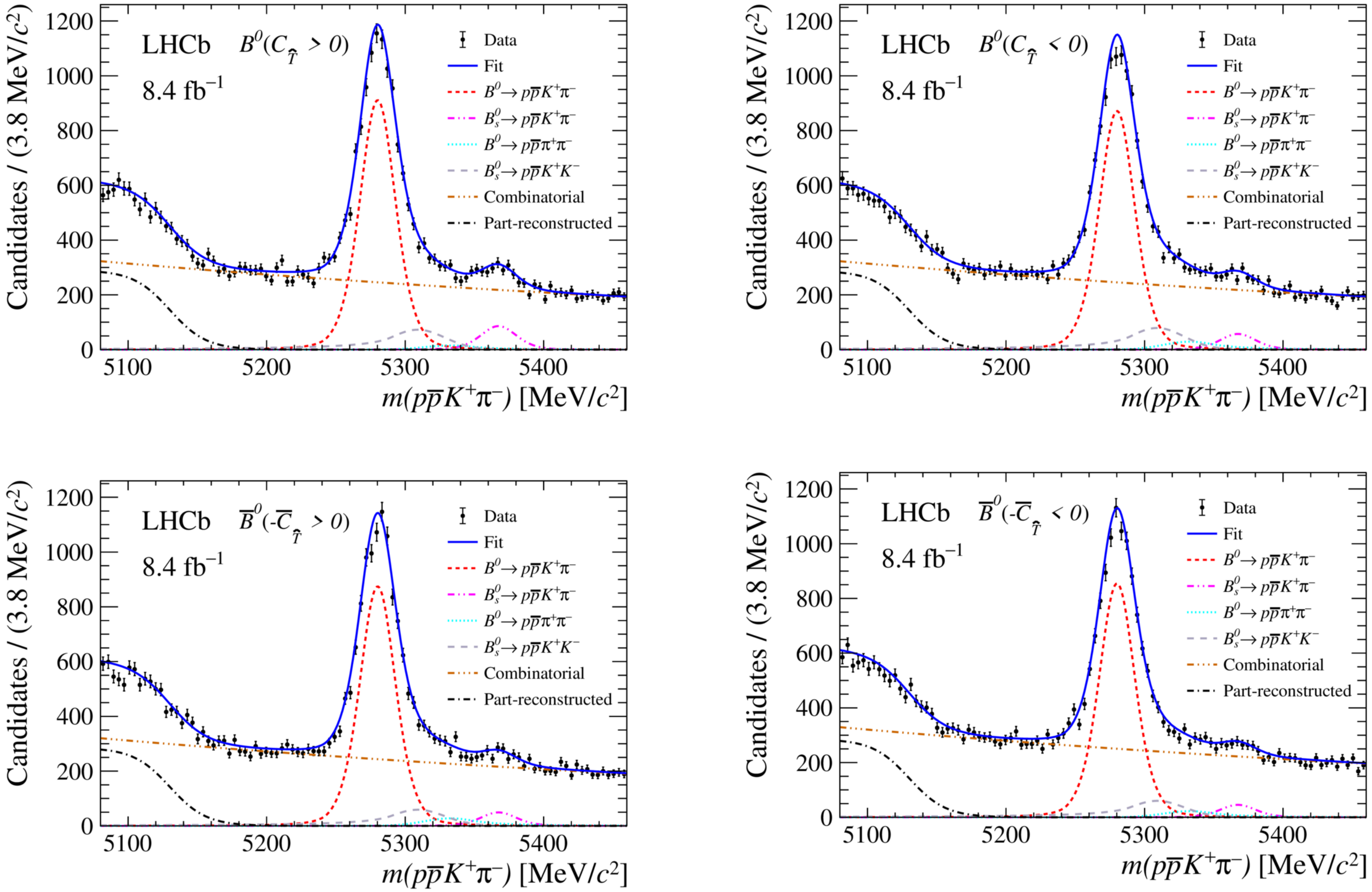
Recent measurements by Belle in red

arXiv:2305.12806v1



T-odd results $B^0 \to p\bar{p}K^+\pi^-$

arXiv:2205.08973v1



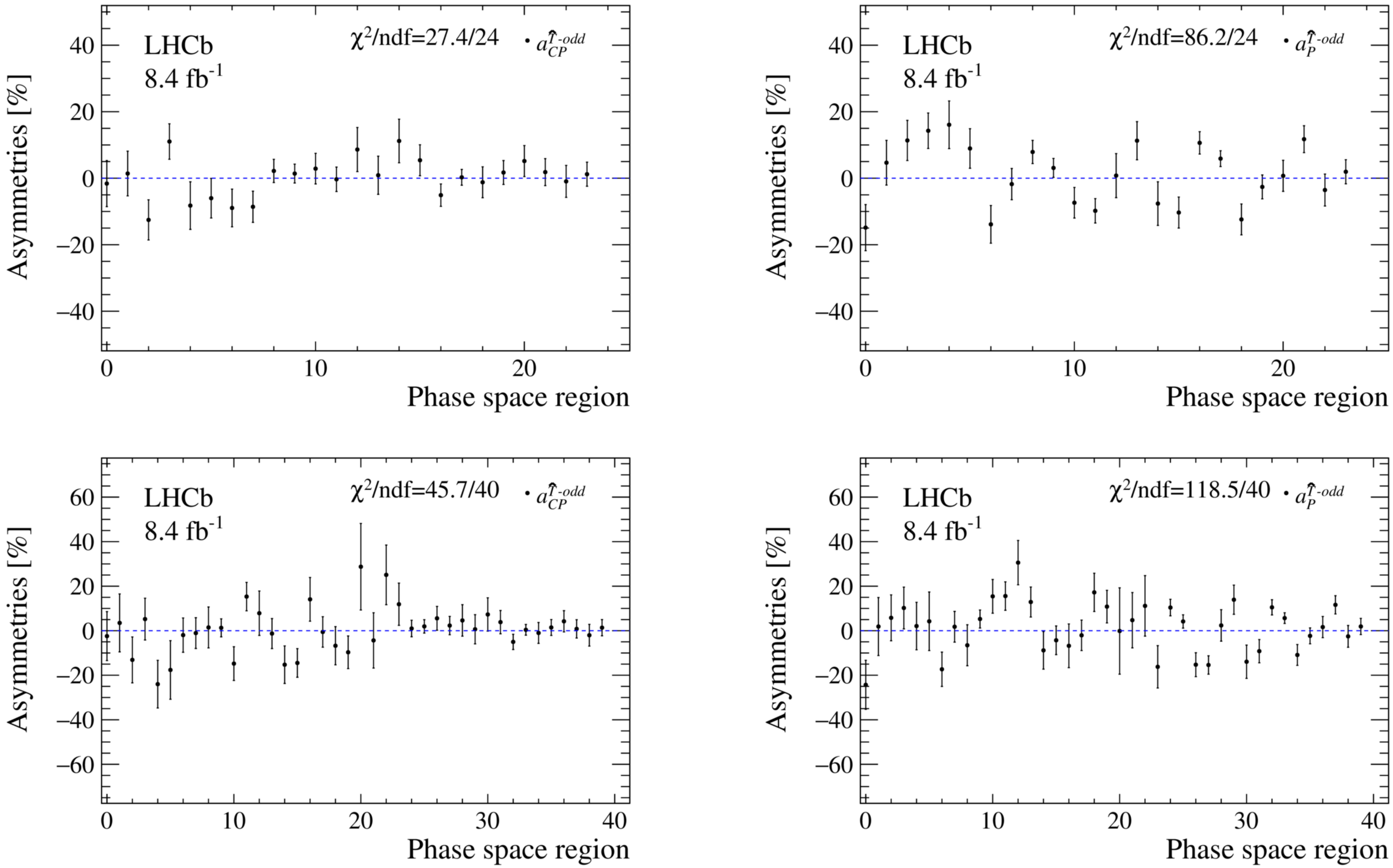
$$C_{\hat{T}} = \vec{p}_{K^+} \cdot (\vec{p}_{\pi^-} \times \vec{p}_p), \quad \bar{C}_{\hat{T}} = \vec{p}_{K^-} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\bar{p}}).$$

$$a_P^{T\text{-odd}} = (1.49 \pm 0.85 \pm 0.08)\%,$$

$$a_{CP}^{\hat{T}\text{-odd}} = (0.51 \pm 0.85 \pm 0.08)\%,$$

In phase space regions

arXiv:2205.08973v1



No CP violation

Observed P violation: χ^2= 86.2/24 (χ^2= 118.5/40) translates to p-value of 6.1×10^{-9}(1.1×10^{-9}), corresponding to 5.8σ (6.0σ) deviation

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Comparing binned Dalitz plots

The Miranda method

IIntroduced by BaBar: PRD78, 051102 (2008). Developed further in PRD 80, 096006 (2009), PRD86, 036005 (2012)

- Divide the Dalitz plot in two-dimensional bins
- Compute, for each bin, the significance of the difference in the numbers of $D^{+}_{(s)}$ candidates and $D^{-}_{(s)}$ candidates, where the latter is corrected for global charge asymmetry (e.g. from production and detection).

$$S^{i}_{CP} = \frac{N^{i}(D^{+}_{(s)}) - \alpha N^{i}(D^{-}_{(s)})}{\sqrt{\alpha(\delta^{2}_{N^{i}(D^{+}_{(s)})} + \delta^{2}_{N^{i}(D^{-}_{(s)})})}} \qquad \alpha = \frac{\sum_{i} N^{i}(D^{+}_{(s)})}{\sum_{i} N^{i}(D^{-}_{(s)})}$$

- Two-sample χ^2 test: calculate p-value for no-CPV hypothesis based on $\chi^{2}(\mathcal{S}_{CP}) = \sum (\mathcal{S}_{CP})^{2}$

Applied also to:

LHCb D→KKπ PRD 84.112008 (2011)

LHCb D→3π PLB 728 (2014) 585-595

CDF D→KSππ PRD 86, 032007 (2012)

LHCb D→φπ, D→KSπ JHEP 1306 (2013) 112

BaBar D→KKπ: PRD 87 (2013) 052010 (check)

LHCb D°→πππ° PLB 740, 158 (2015).

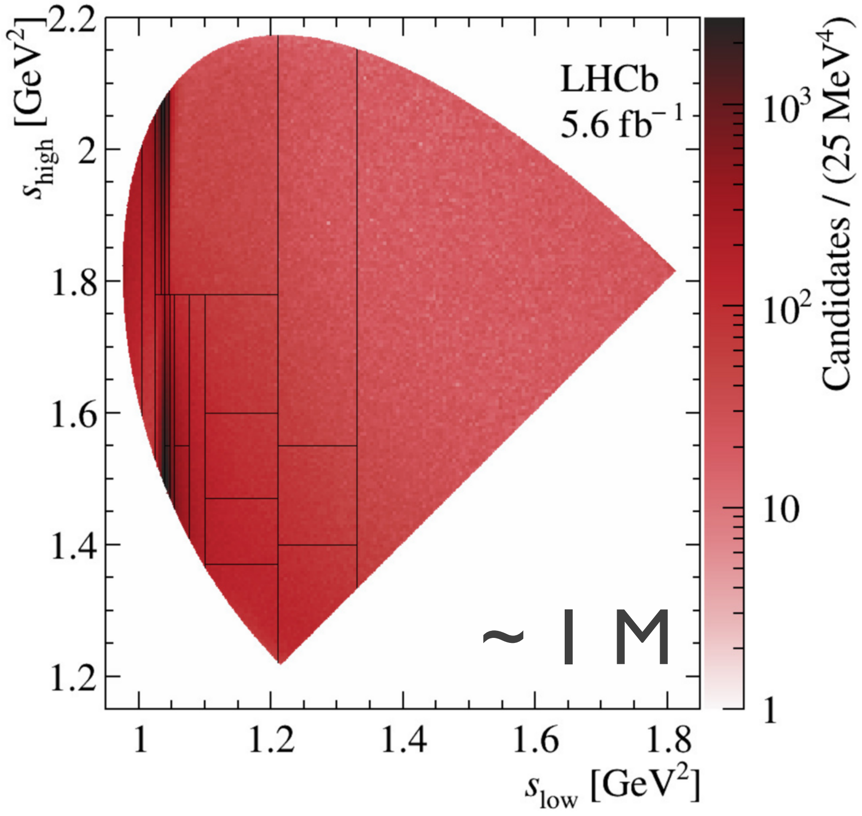
LHCb D→KKππ, D→4π PLB 726 (2013) 623-633 (5D bins)

Search for CP violation in $D^+_{(s)} \to K^-K^+K^+$

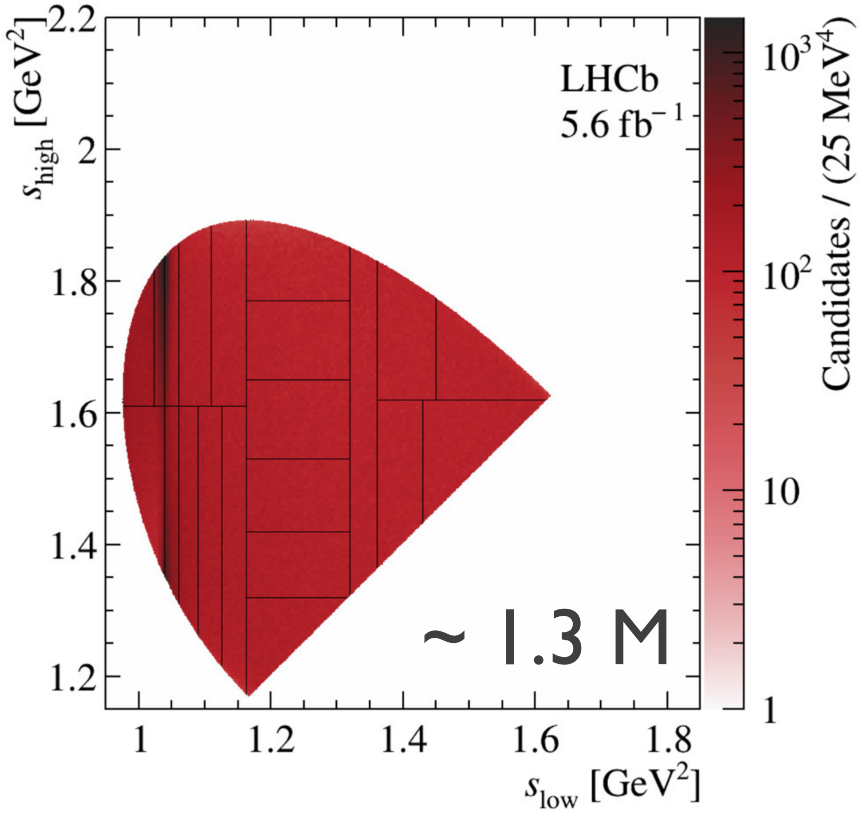
- Singly Cabibbo suppressed $D^+_{(s)}$ decays
- Signal purity 64% (D^+_s) and 78% (D^+)
- LHCb Run 2 data (5.6 fb^{-1})
-

21 bins in total overlaid

$D^+_s \to K^-K^+K^+$



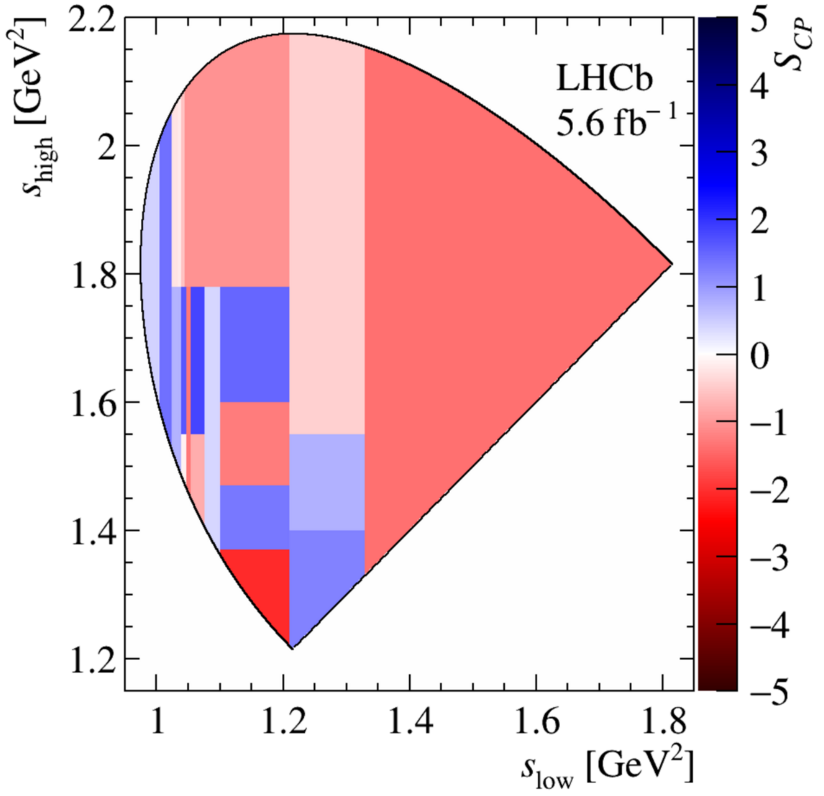
$D^+ \to K^-K^+K^+$

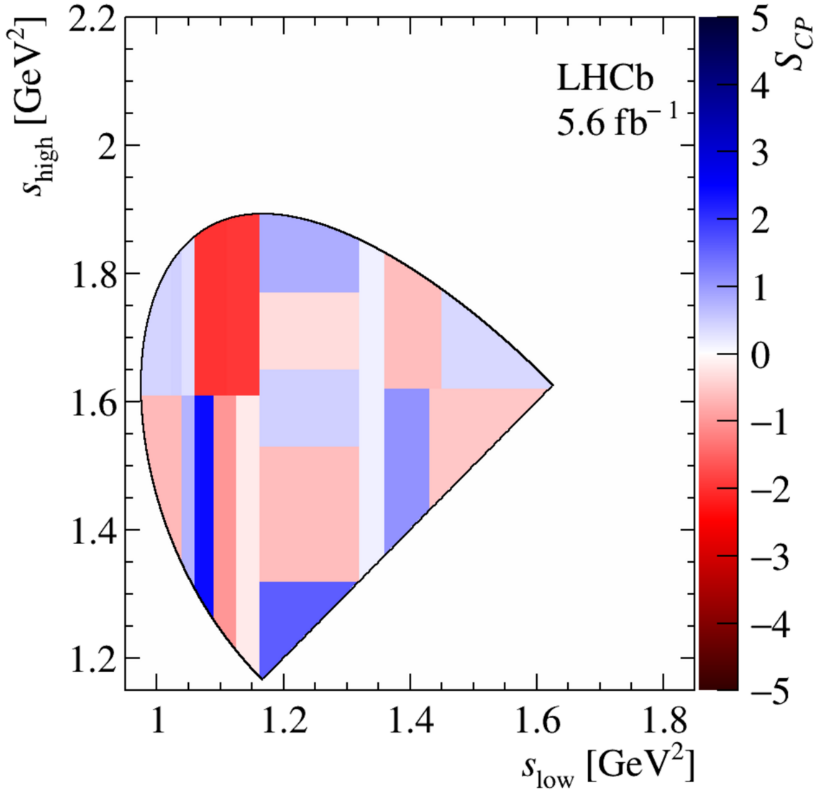


Modified Miranda: Fit in each bin, no background (fit per bin)

Results

- Control samples:
  - Phase space simulation
  - Background samples
  - $D^+_s \to K^-K^+\pi^+$ and $D^+ \to K^-\pi^+\pi^+$ (CF)
- Stability checks:
  - different invariant mass fit models
  - different binning schemes
- No evidence for CP violation
  - p-value ($D^+_s \to K^-K^+K^+$) = 13.3%
  - p-value ($D^+ \to K^-K^+K^+$) = 31.6%





CP violation in $B^{\pm} \to hhh$

- Quite complex

- Short- and long-distance contributions to the generation of the strong-phase differences

- A recent amplitude analysis found a large CP asymmetry related to the interference between the S- and P-wave contributions in $B^+ \to \pi^+\pi^+\pi^-$ decays, also in S- wave and in D- wave [Phys. Rev. D101, (2020) 012006; Phys. Rev. Lett. 124, (2020) 031801]

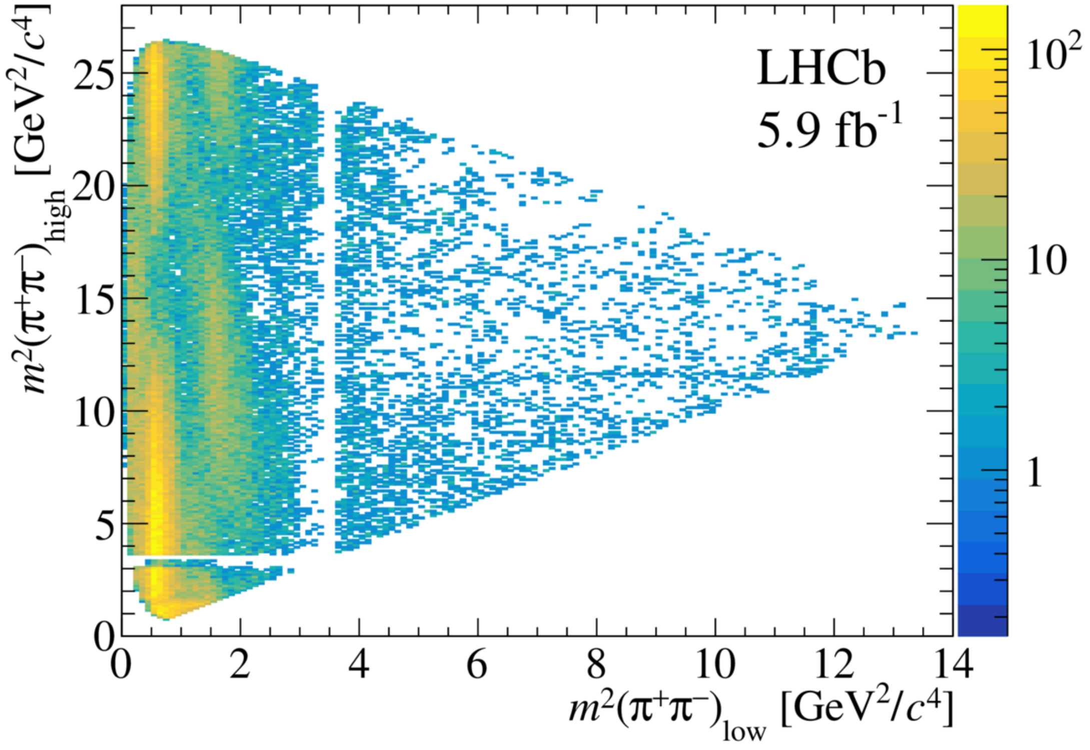
- CP violation involving ππ → KK rescattering was observed in $B^+ \to \pi^+K^+K^-$ decays : A_{CP}= (–66.4 °± 3.8 (stat) ±° 1.9 (syst))% [Phys. Rev. Lett. 123, (2019) 231802]

- The patterns of localised CP asymmetries in the four charmless decay modes $B^+ \to \pi^+\pi^+\pi^-$, $B^+ \to K^+K^+K^-$, $B^+ \to \pi^+K^+K^-$ and $B^+ \to K^+\pi^+\pi^-$ can be interpreted as originating from long-distance hadronic interactions

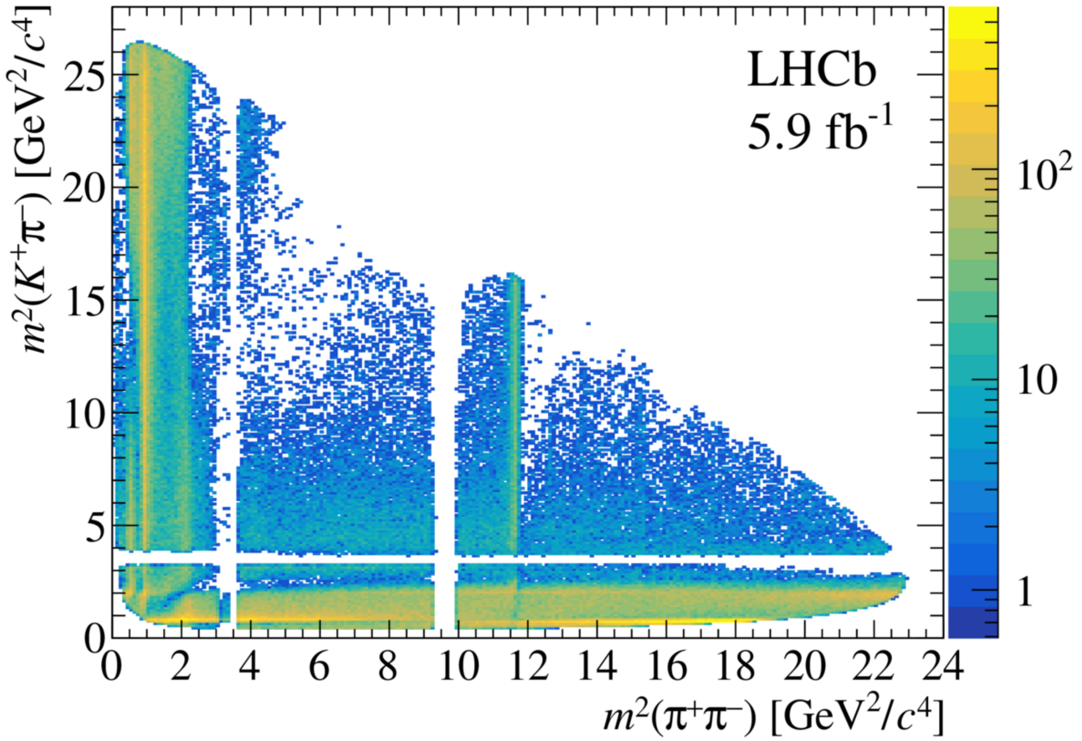
Phase space distributions

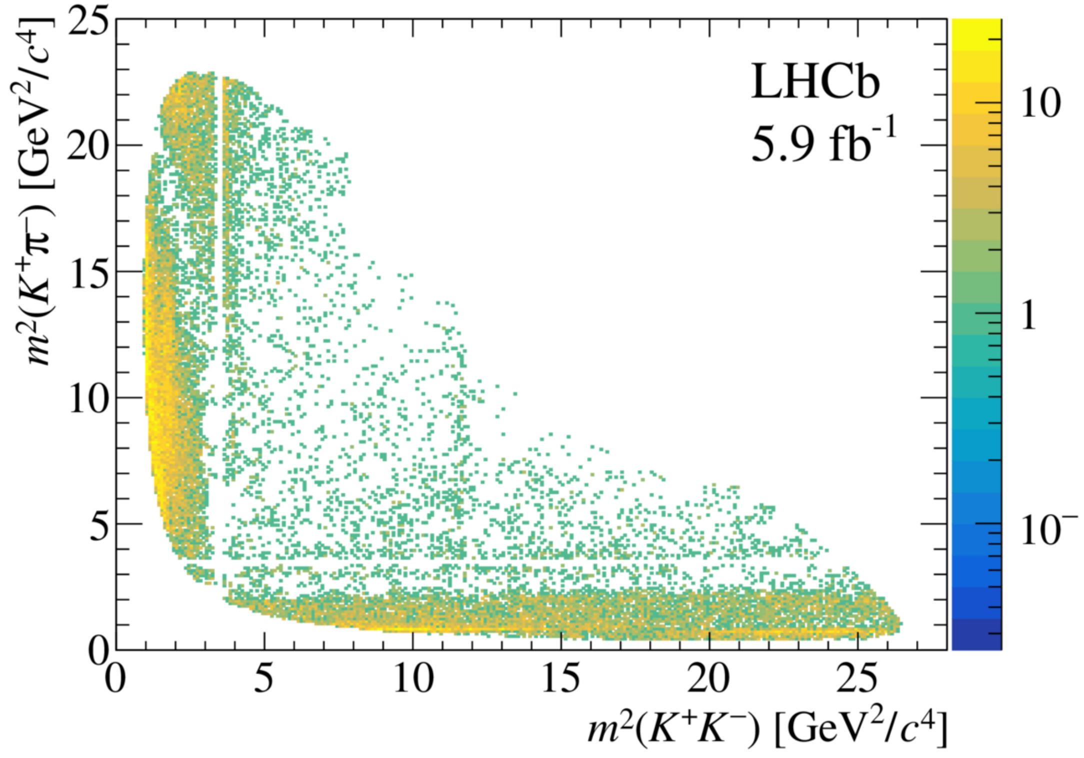
arXiv:2206.07622

$B^+ \rightarrow \pi^+\pi^+\pi^-$

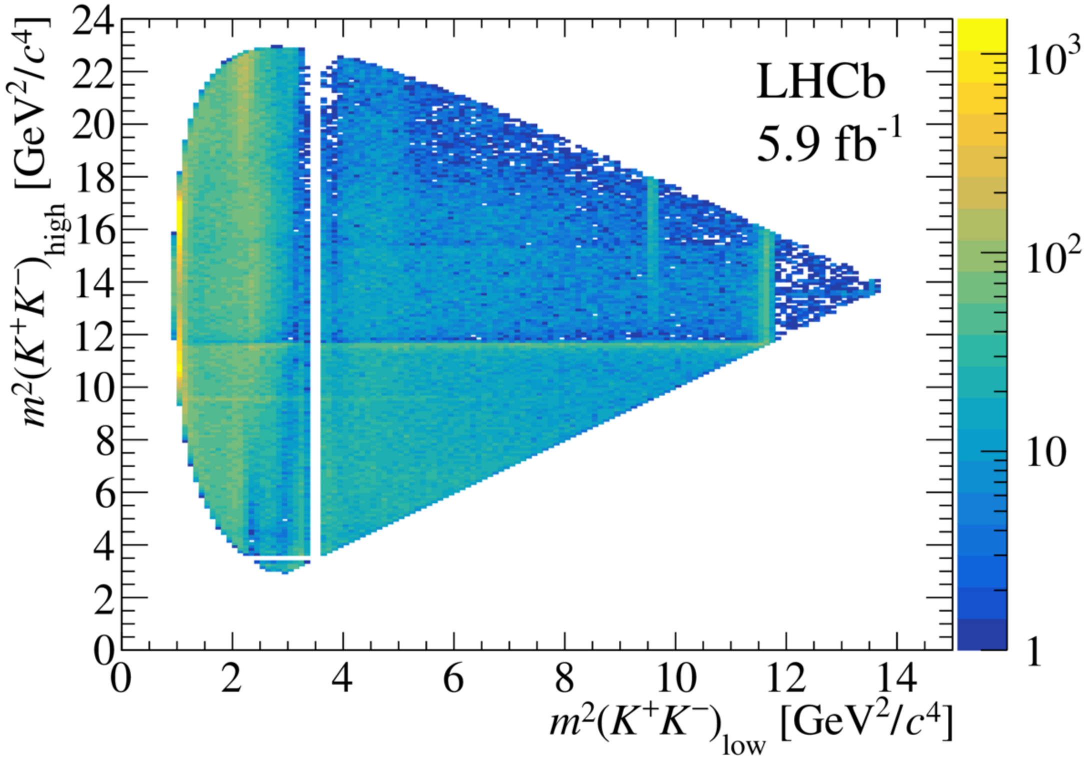


$B^+ \rightarrow K^+\pi^+\pi^-$





$B^+ \rightarrow \pi^+K^+K^-$



$B^+ \rightarrow K^+K^+K^-$

Binned asymmetries

- Split by flavour
- Subtract the background, correct for efficiencies
- Calculate asymmetries per bin

$$A_{CP}(s_{12}, s_{23}) = \left(\frac{d\Gamma(\bar{P} \to \bar{f})}{d\Omega} - \frac{d\Gamma(P \to f)}{d\Omega}\right) \Big/ \left(\frac{d\Gamma(\bar{P} \to \bar{f})}{d\Omega} + \frac{d\Gamma(P \to f)}{d\Omega}\right)$$

- Adaptive binning: with approximately the same number of events
  - $B^\pm \to \pi^\pm\pi^+\pi^-$: 400 bins ~ 229 events/bin
  - $B^\pm \to K^\pm\pi^+\pi^-$: 1728 bins ~ 276 events/bin
  - $B^\pm \to \pi^\pm K^+K^-$: 256 bins ~ 127 events/bin
  - $B^\pm \to K^\pm K^+K^-$: 729 bins ~ 461 events/bin

arXiv:2206.07622

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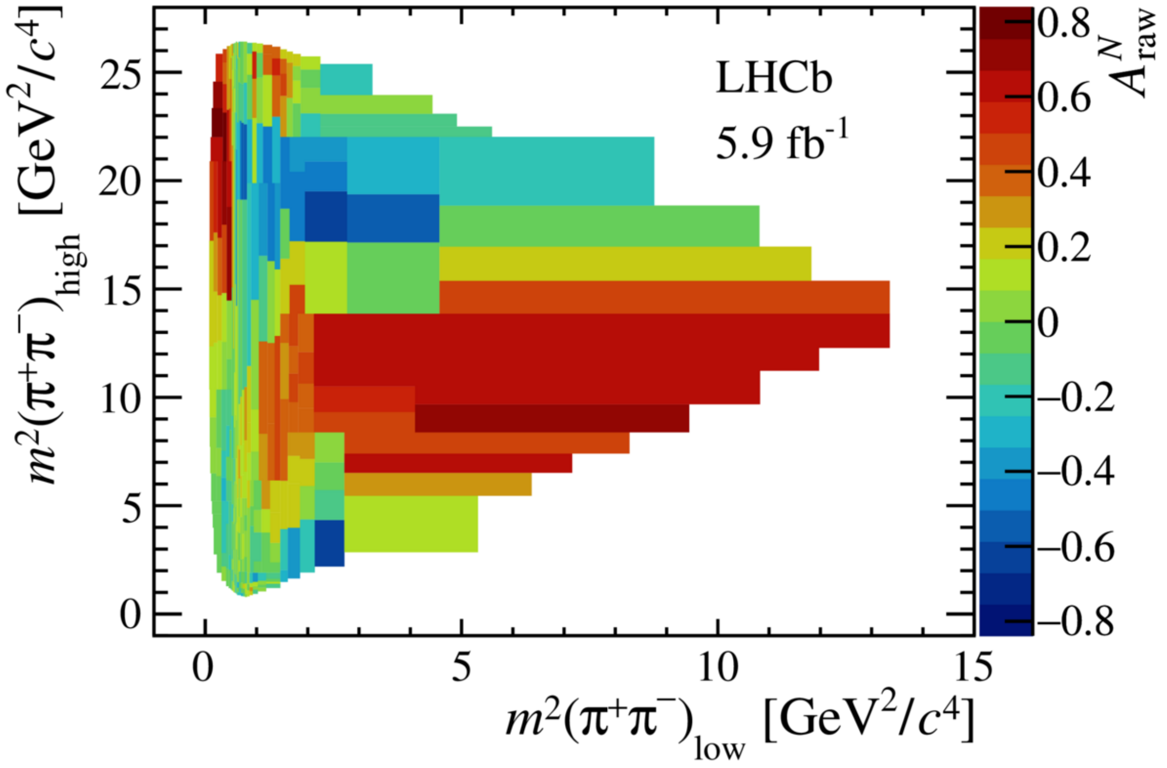
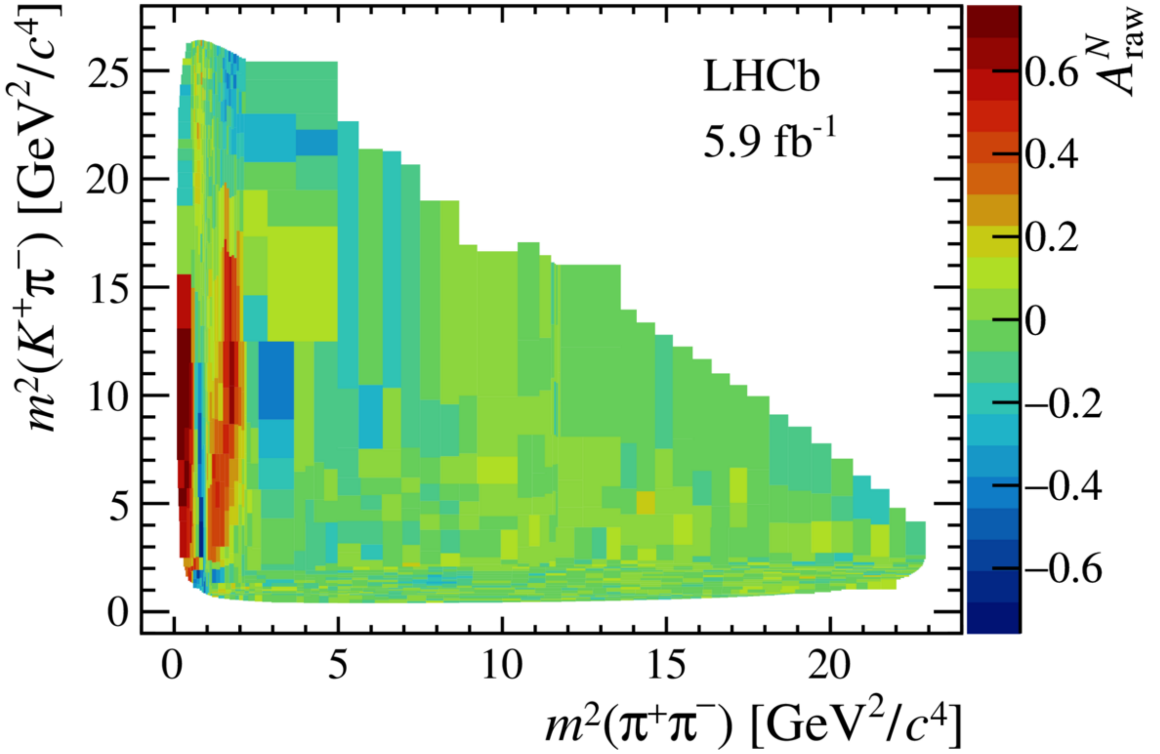
Local asymmetries

arXiv:22

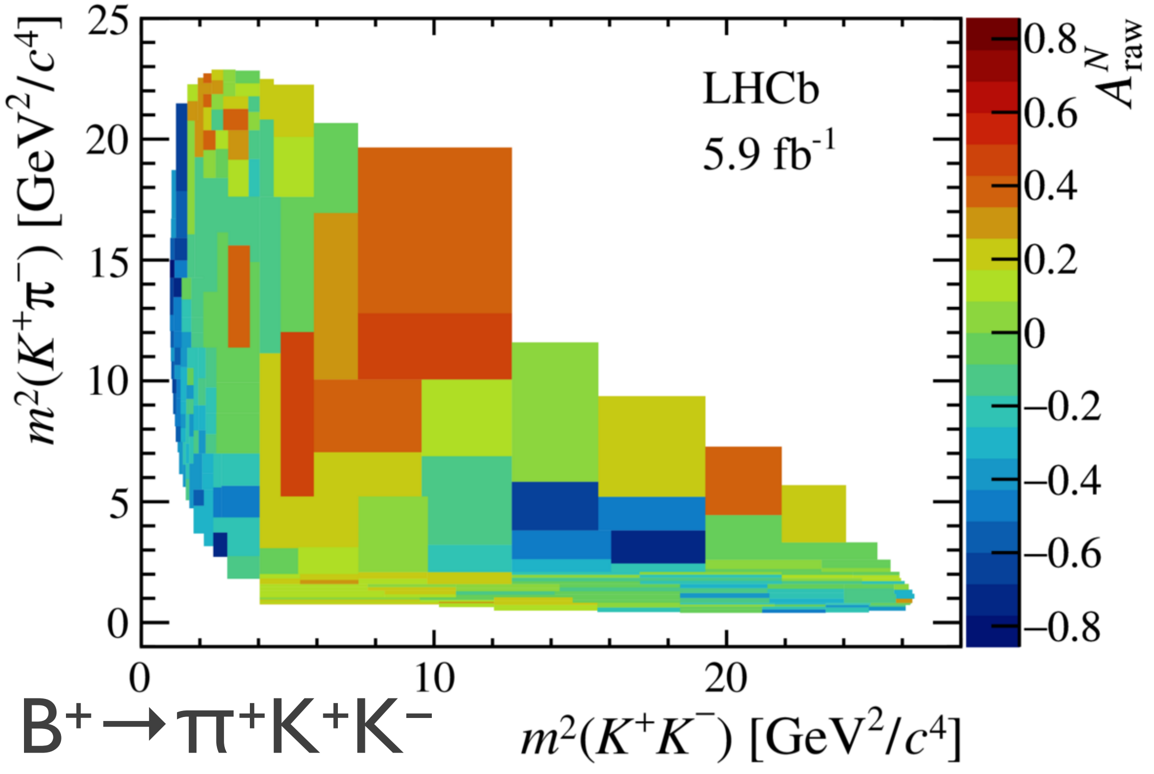
$B^+ \to \pi^+\pi^+\pi^-$



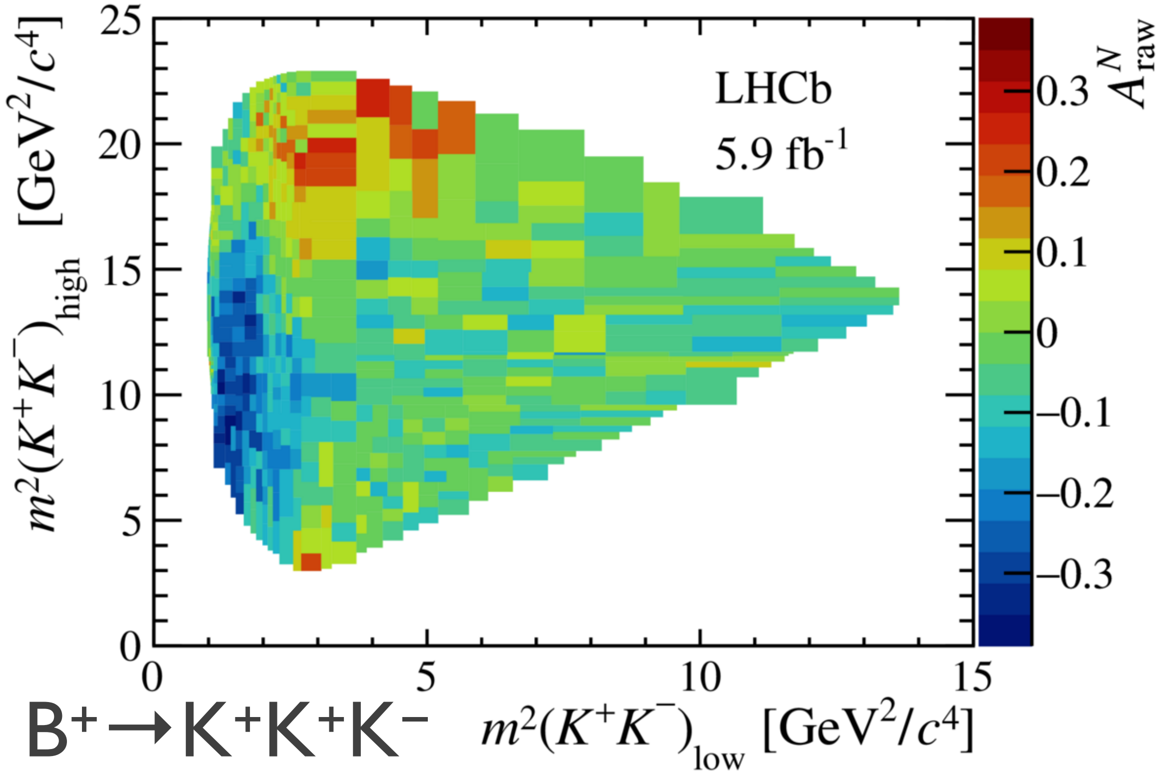
$B^+ \to K^+\pi^+\pi^-$



$B^+ \to \pi^+K^+K^-$



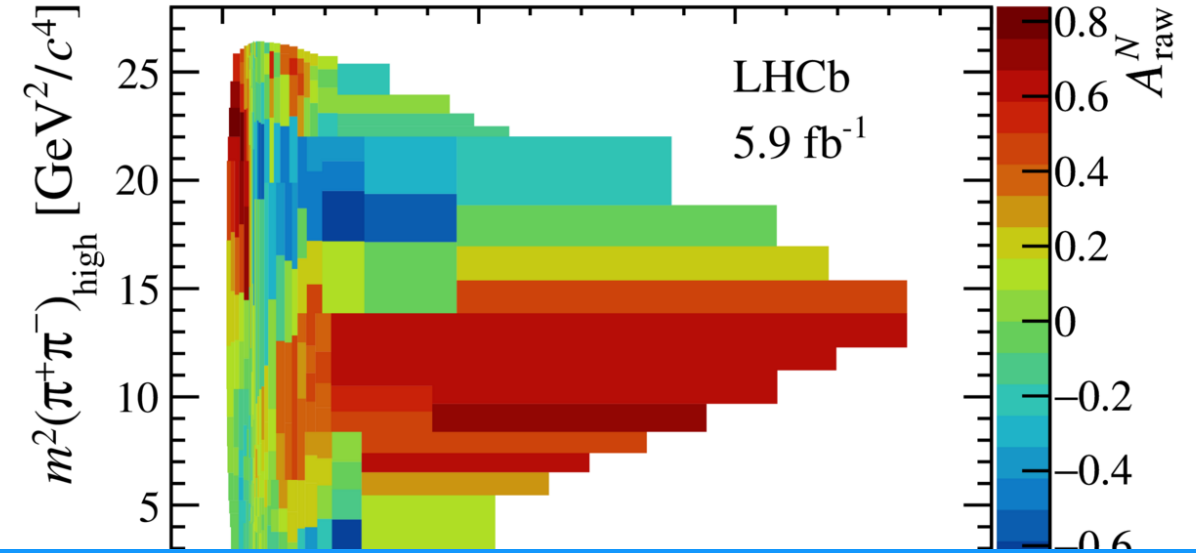
$B^+ \to K^+K^+K^-$



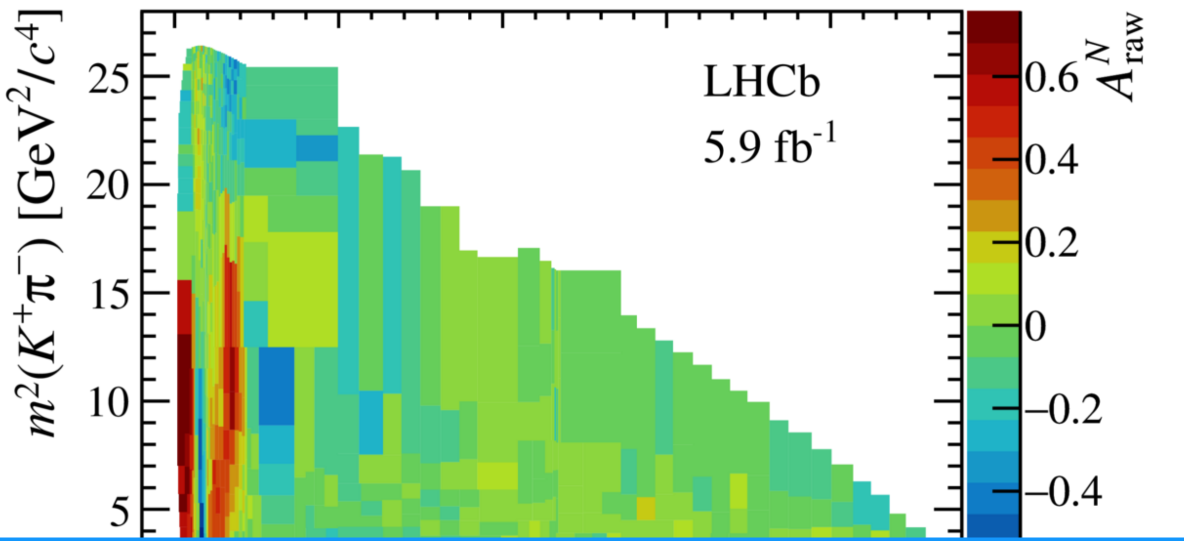
Local asymmetries

arXiv:22

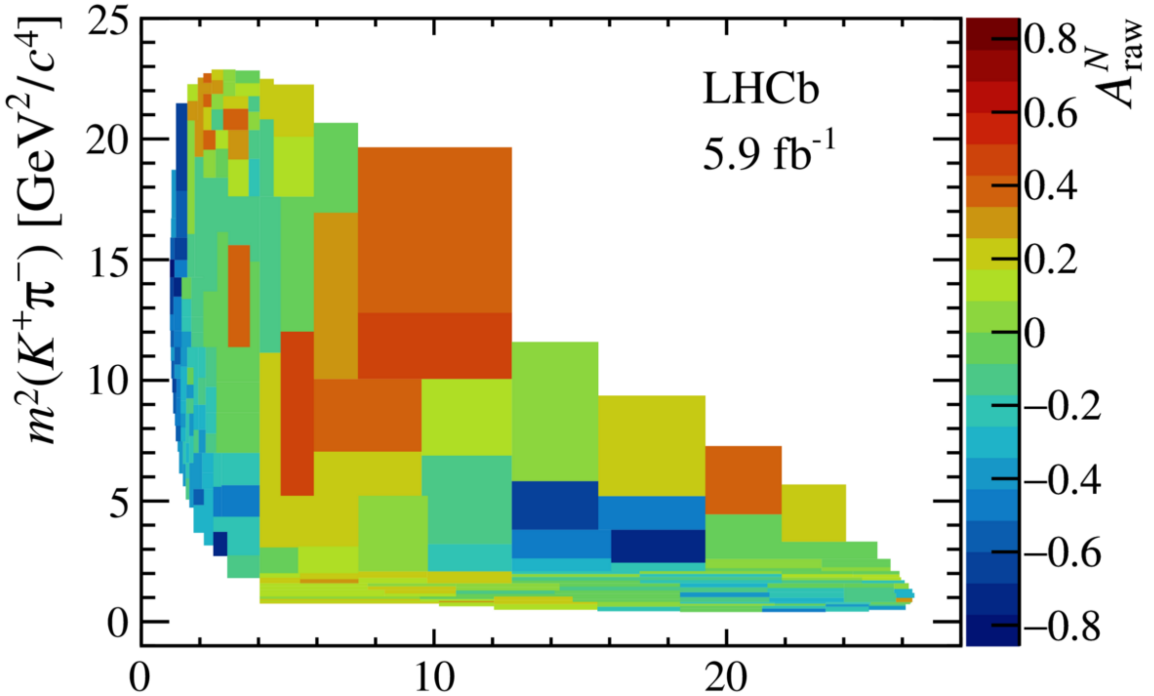
B⁺→π⁺π⁺π⁻



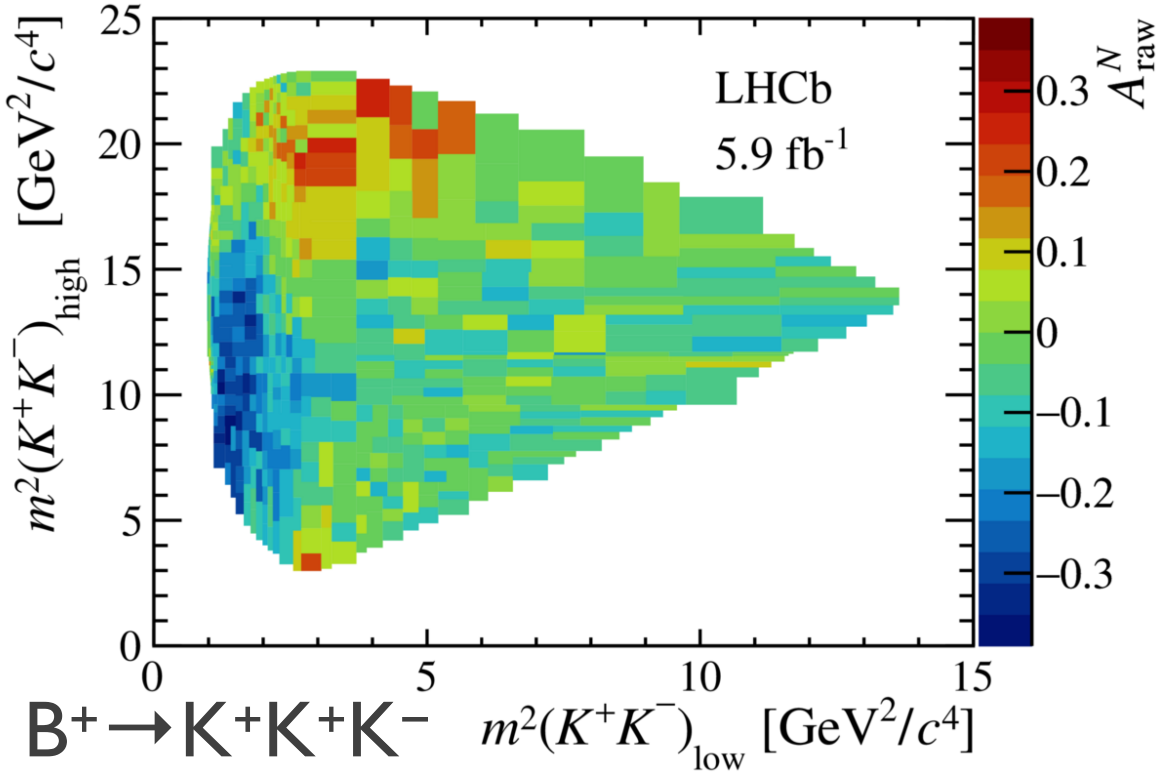
B⁺→K⁺π⁺π⁻



Suitable for computing lab for training undergrad students (with simplifications and small open data set from Run 1)



B⁺→π⁺K⁺K⁻



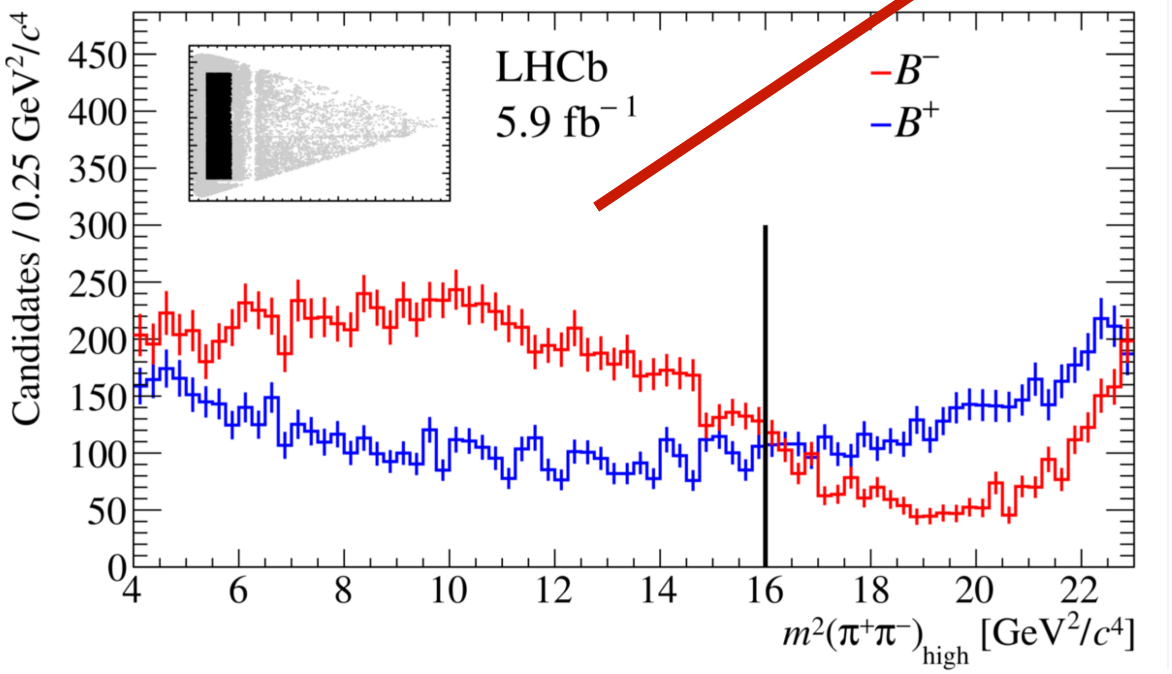
B⁺→K⁺K⁺K⁻

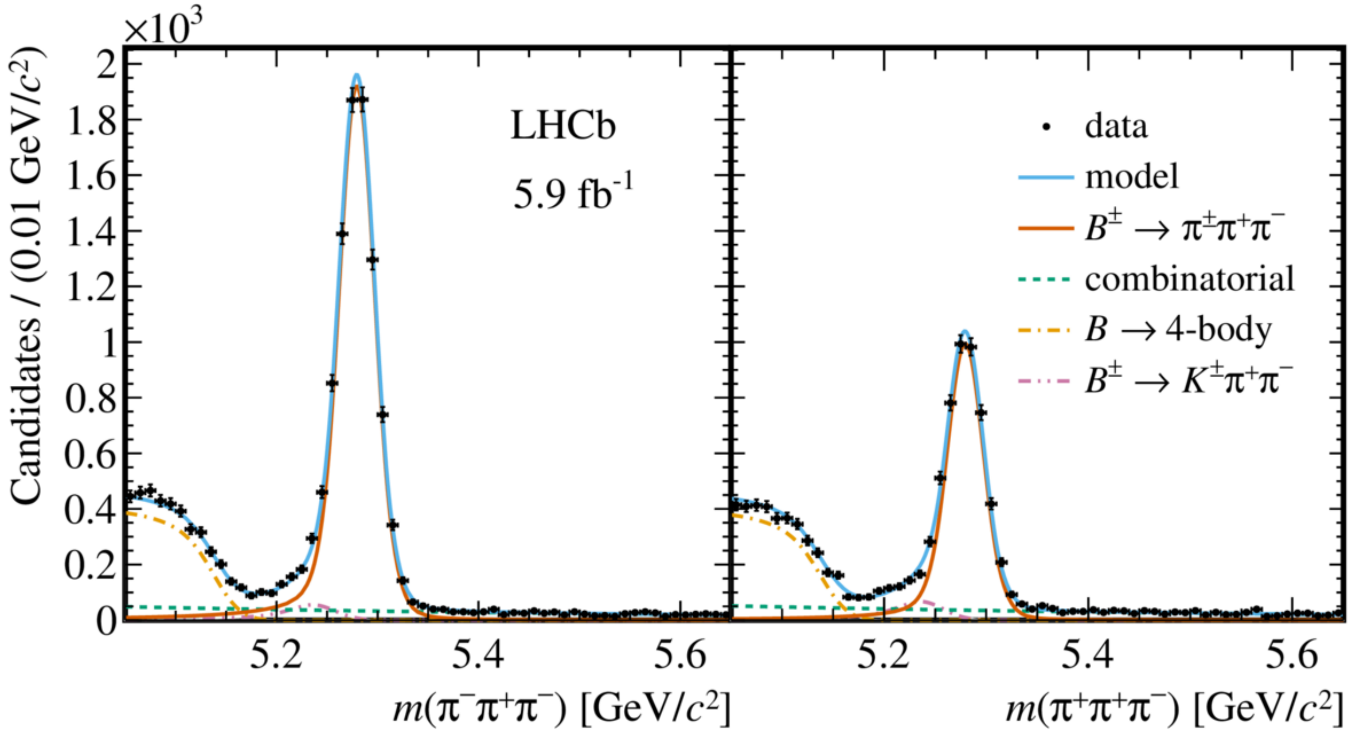
Local asymmetries and rescattering

- Rich pattern of large and localised asymmetries which result from interference between the contributions
- Possible ππ → KK rescattering
- The rescattering region is defined in the Dalitz plot in the two-kaon or two-pion invariant mass range 1.1–2.25 GeV^2/c^4 for $B^+ \rightarrow K^+K^+K^-$ due to the presence of the ϕ(1020) resonance and 1–2.25 GeV^2/c^4 for the other three channels

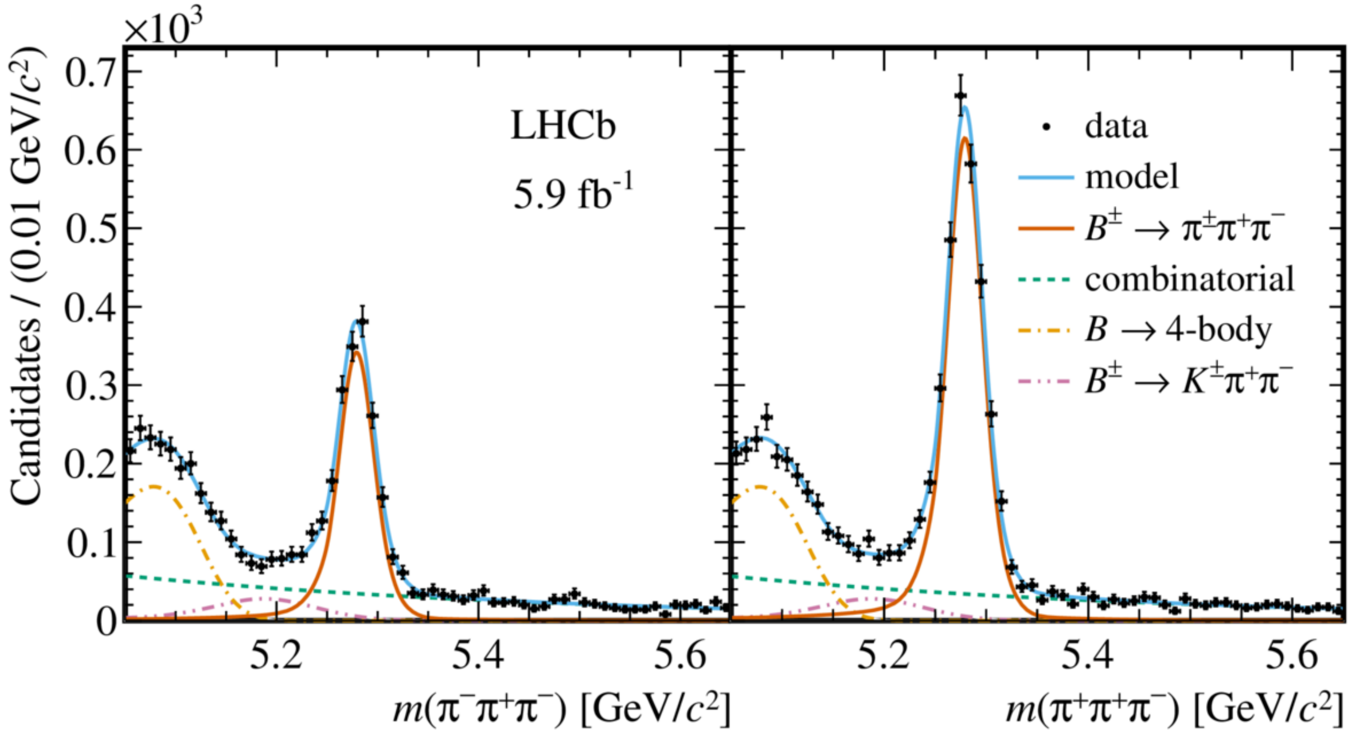
Rescattering region $B^+ \to \pi^+\pi^+\pi^-$

arXiv:2206.07622





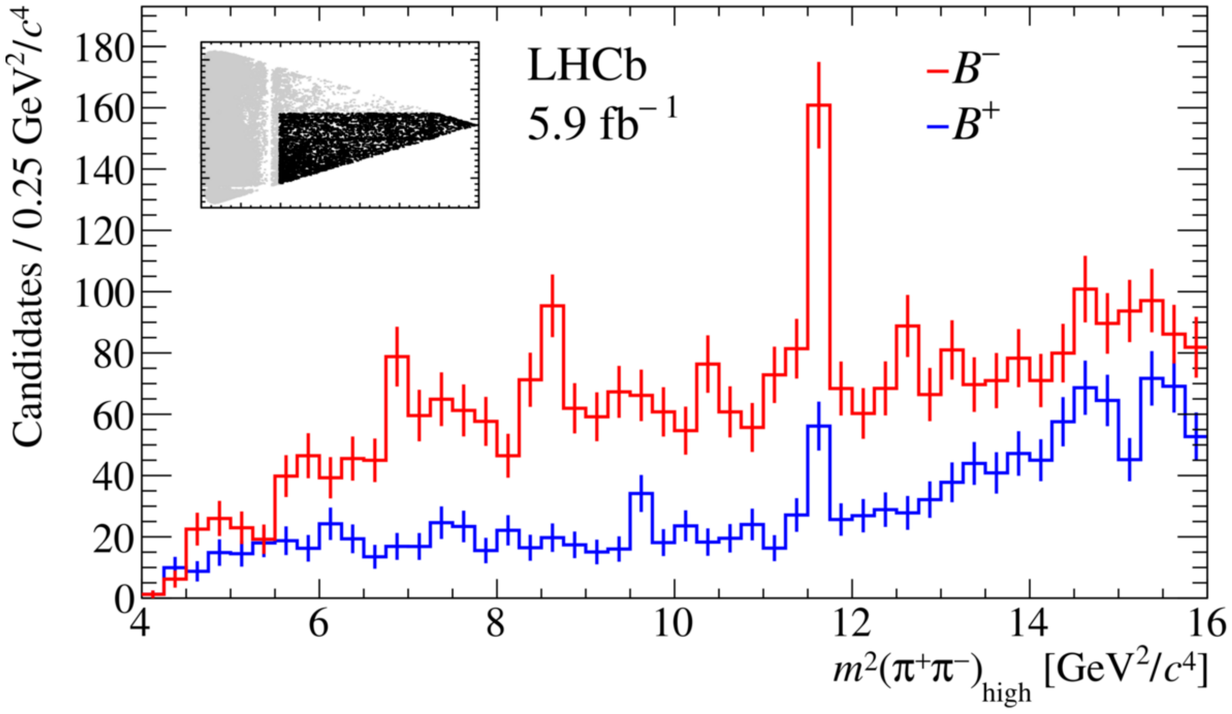
flip of the asymmetry between the two regions

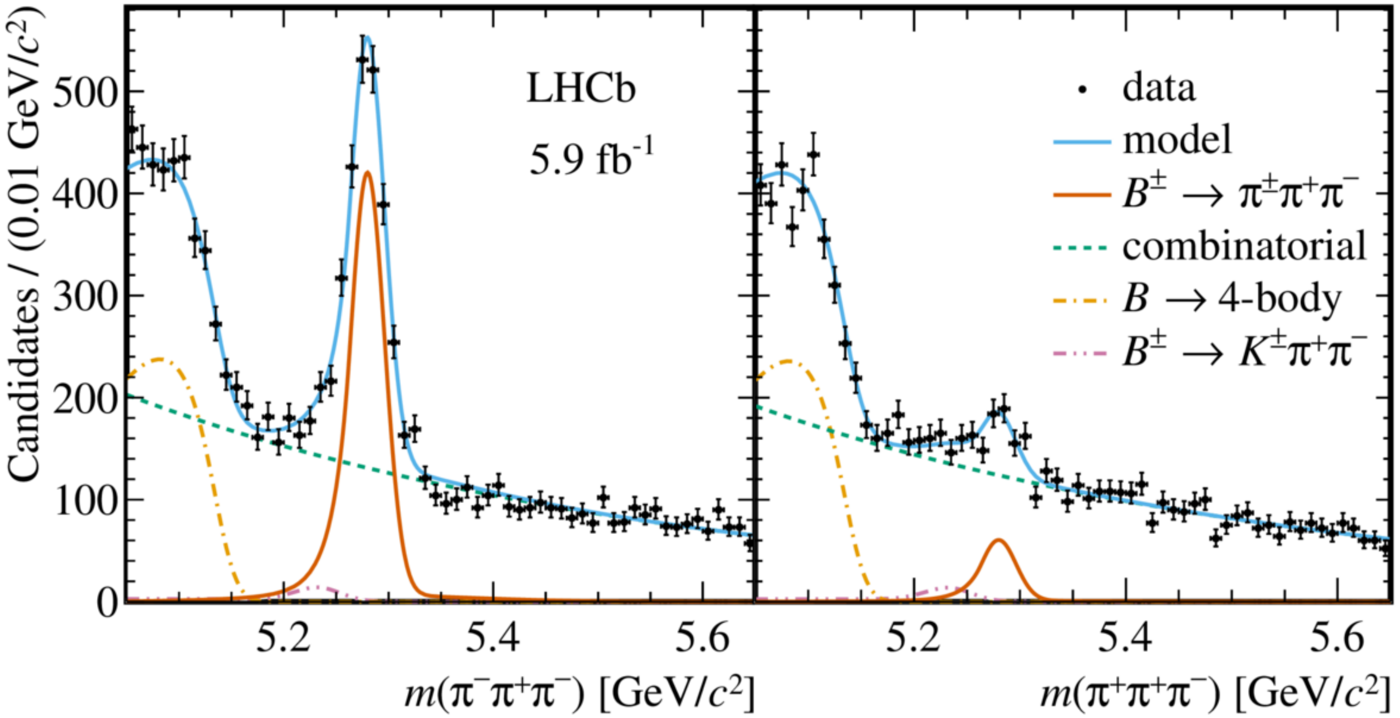


Higher mass region $B^+ \to \pi^+\pi^+\pi^-$

arXiv:2206.07622

- Higher mass region
- Clear $\chi_{c0}(1P)$ contribution
  - comes from the B decay, as no such structure was observed in the invariant mass sidebands
- Large asymmetry is observed





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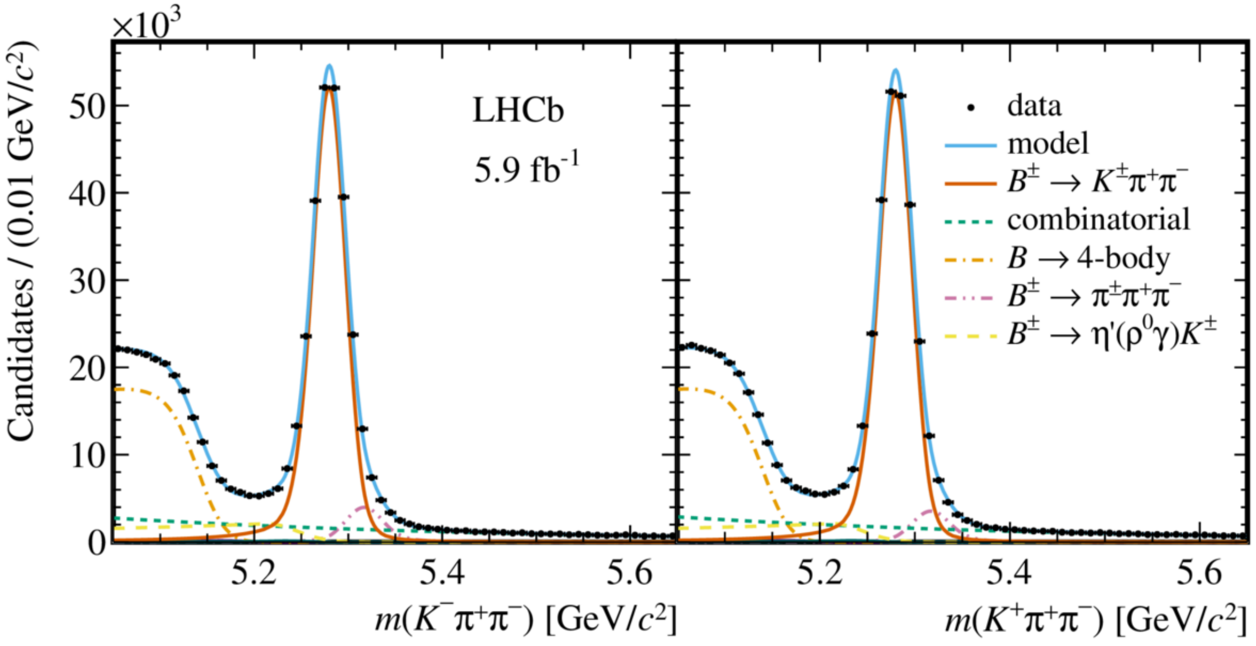
Integrated asymmetries

Direct CP violation in B±→hhh

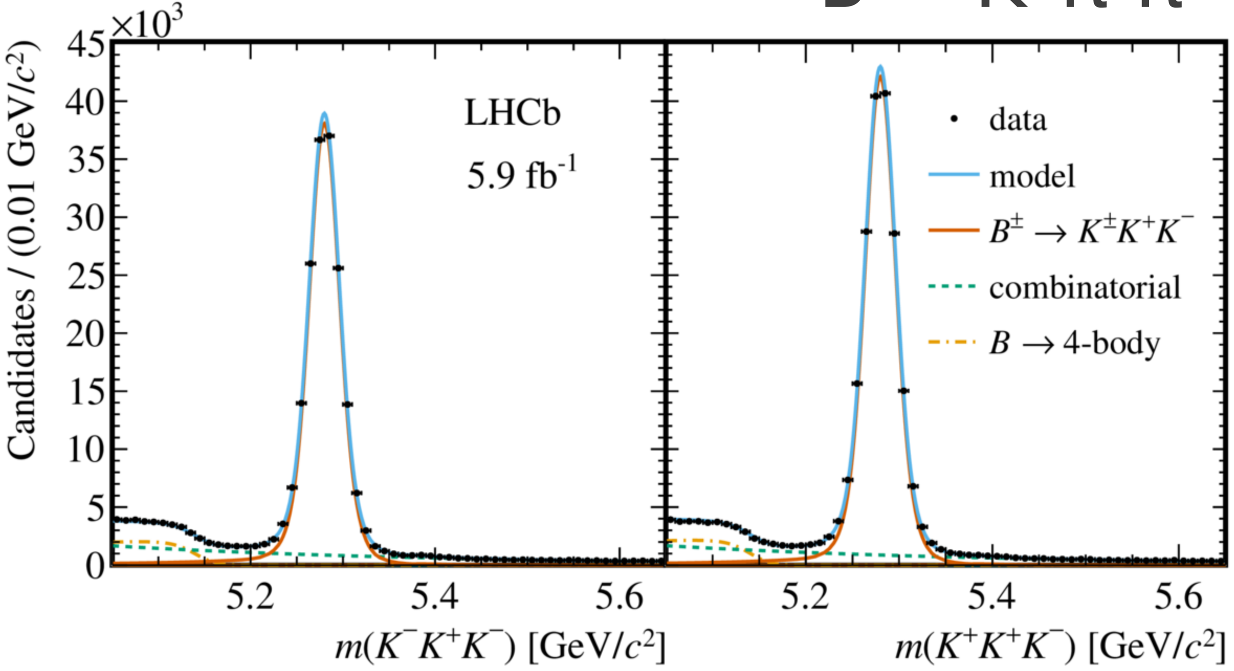
phase-space-integrated CP asymmetries

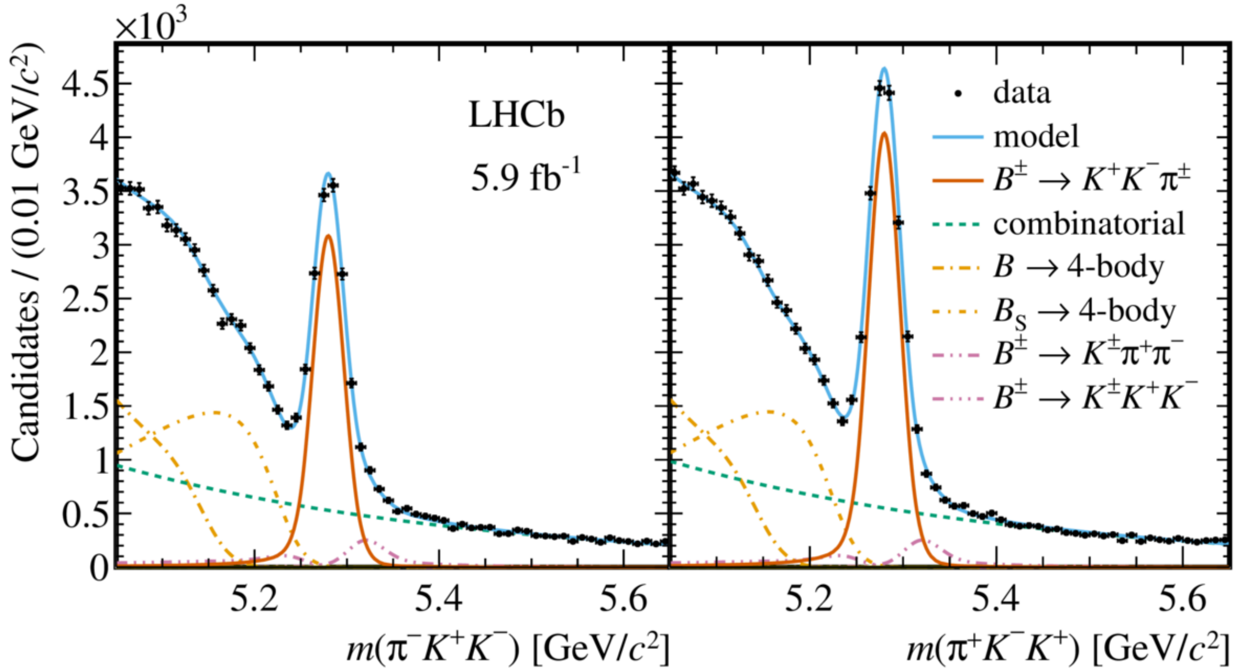
arXiv:2206.07622

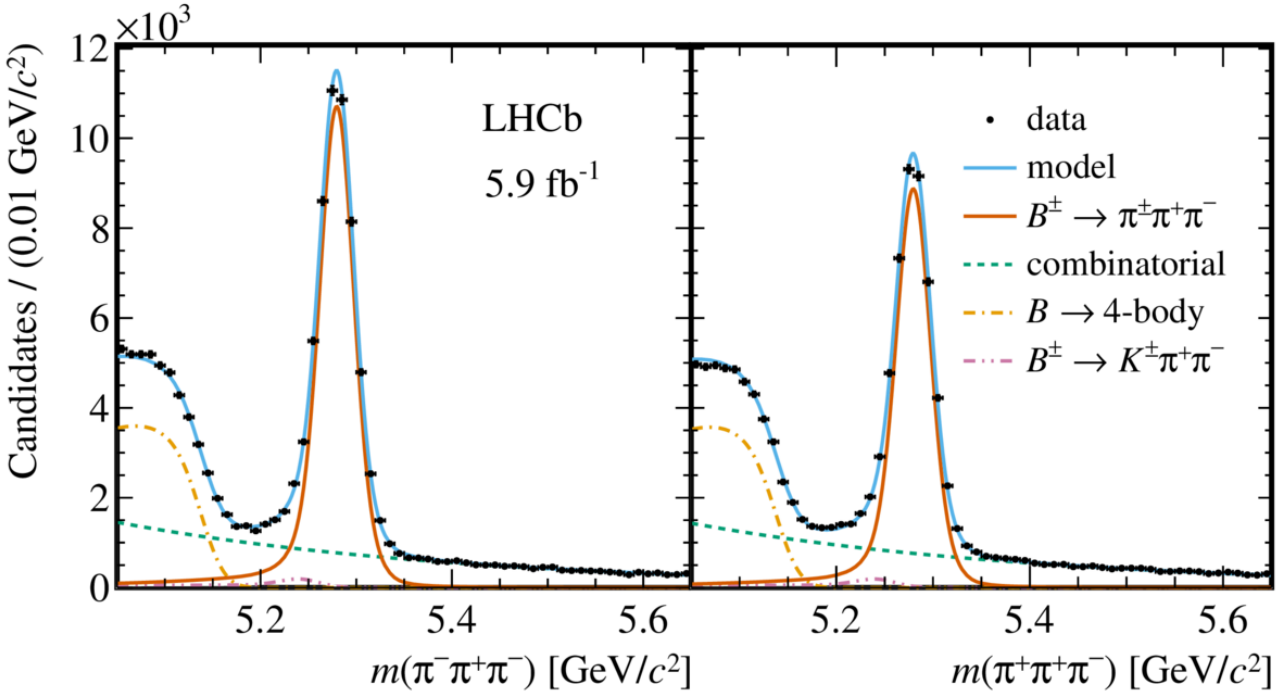
$B^+ \to \pi^+\pi^+\pi^-$



$B^+ \to K^+\pi^+\pi^-$







$B^+ \to \pi^+K^+K^-$

$B^+ \to K^+K^+K^-$

Integrated asymmetry results

arXiv:2206.07622

- The CP asymmetry

$$A_{CP} = \frac{A_{raw}^{corr} - A_P}{1 - A_{raw}^{corr} A_P}$$

Production asymmetry

Efficiency corrected raw asymmetry

- Significant inclusive CP asymmetries are found for the latter three B decay channels, two observed for the first time

$$\begin{aligned}
A_{CP}(B^\pm \to K^\pm\pi^+\pi^-) &= +0.011 \pm 0.002 \pm 0.003 \pm 0.003,\\
A_{CP}(B^\pm \to K^\pm K^+ K^-) &= -0.037 \pm 0.002 \pm 0.002 \pm 0.003,\\
A_{CP}(B^\pm \to \pi^\pm\pi^+\pi^-) &= +0.080 \pm 0.004 \pm 0.003 \pm 0.003,\\
A_{CP}(B^\pm \to \pi^\pm K^+ K^-) &= -0.114 \pm 0.007 \pm 0.003 \pm 0.003,
\end{aligned}$$

Also, results on U-spin asymmetries in arXiv:2206.07622

B± → hhh and B → PV

arXiv:2206.02038

- The method that does not require full amplitude analyses
- The method is based on three key features of three-body B decays:
  - the large phase space
  - the dominance of scalar and vector resonances with masses below or around 1 GeV/c^2 (confirmed by amplitude analyses performed by Belle, BaBar and LHCb)
  - the clear signatures of the resonant amplitudes in the Dalitz plot
- Large phase space of these B-meson decays, different types of resonant contributions are allowed

The method for $B \rightarrow PV$

- Start with the decay $B^{\pm} \rightarrow R(\rightarrow h^{-}_{1} h^{+}_{2}) h^{\pm}_{3}$:
  - R is a intermediate resonance
  - $s_{\parallel} = m^2(h^{-}_{1} h^{+}_{2})$ and $s_{\perp} = m^2(h^{-}_{1} h^{\pm}_{3})$
- The resonance line shape (typically a Breit-Wigner distribution) is observed in the projection of the Dalitz plot onto the $s_{\parallel}$ axis
- When a narrow interval in $s_{\parallel}$ around the resonance mass is selected, the projection of the data onto $s_{\perp}$ reflects the angular distribution of the decay products.

The method for B→PV (continued)

arXiv:2206.02038

- Start with the decay $B^{\pm} \rightarrow R(\rightarrow h^{-}_{1} h^{+}_{2}) h^{\pm}_{3}$:
  - R is a intermediate resonance
  - $s_{\parallel} = m^2(h^{-}_{1} h^{+}_{2})$ and $s_{\perp} = m^2(h^{-}_{1} h^{\pm}_{3})$
- In vector resonances, a parabolic shape is observed, since the decay width is proportional to cosine squared of the helicity angle, $\cos^2\theta$, where θ is defined as the angle between h^{-}_{1} and $h^{\pm}_{3}$ computed in the $(h^{-}_{1} h^{+}_{2})$ rest frame.
- If the $(h^{-}_{1} h^{+}_{2})$ pair forms a scalar resonance, the distribution in $s_{\perp}$ is uniform, since the decay of scalar resonances is isotropic in $\cos\theta$.

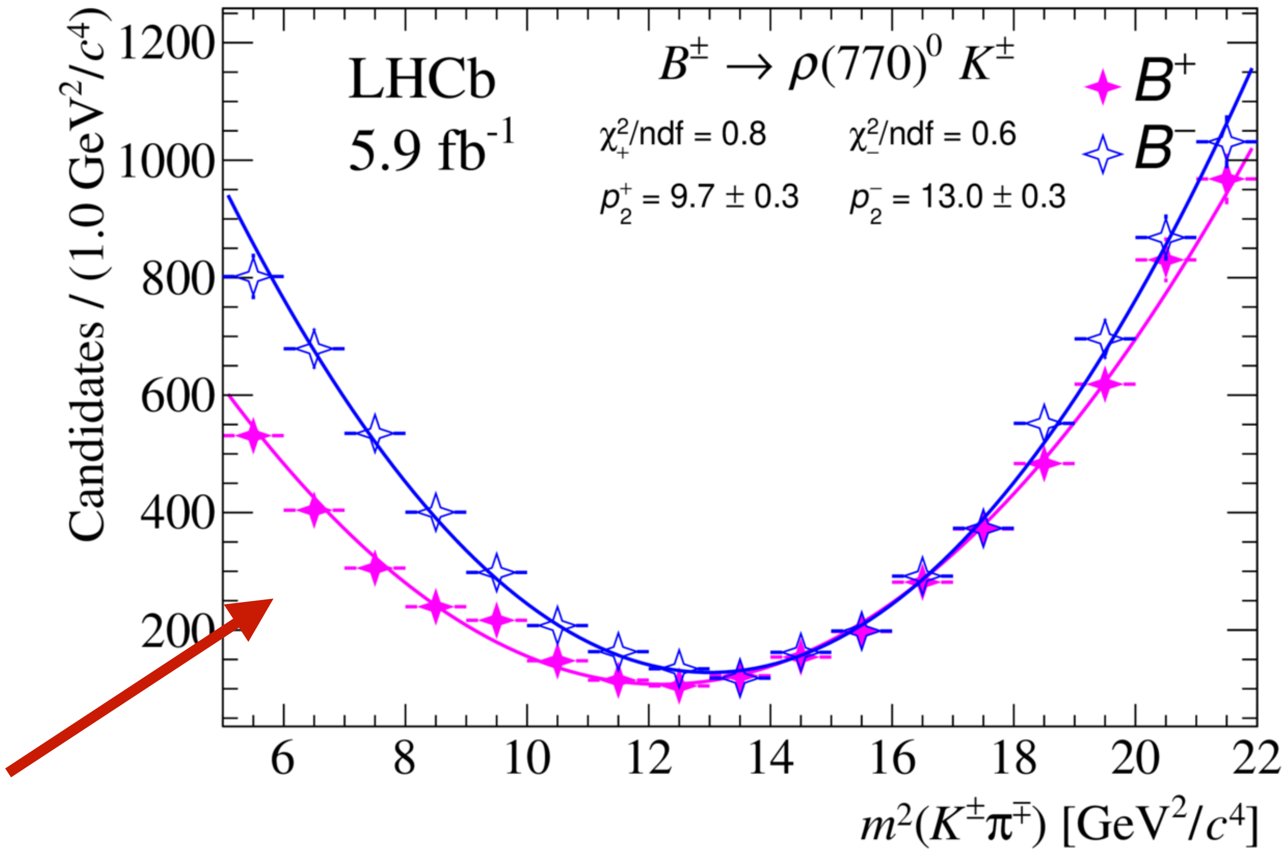
Results

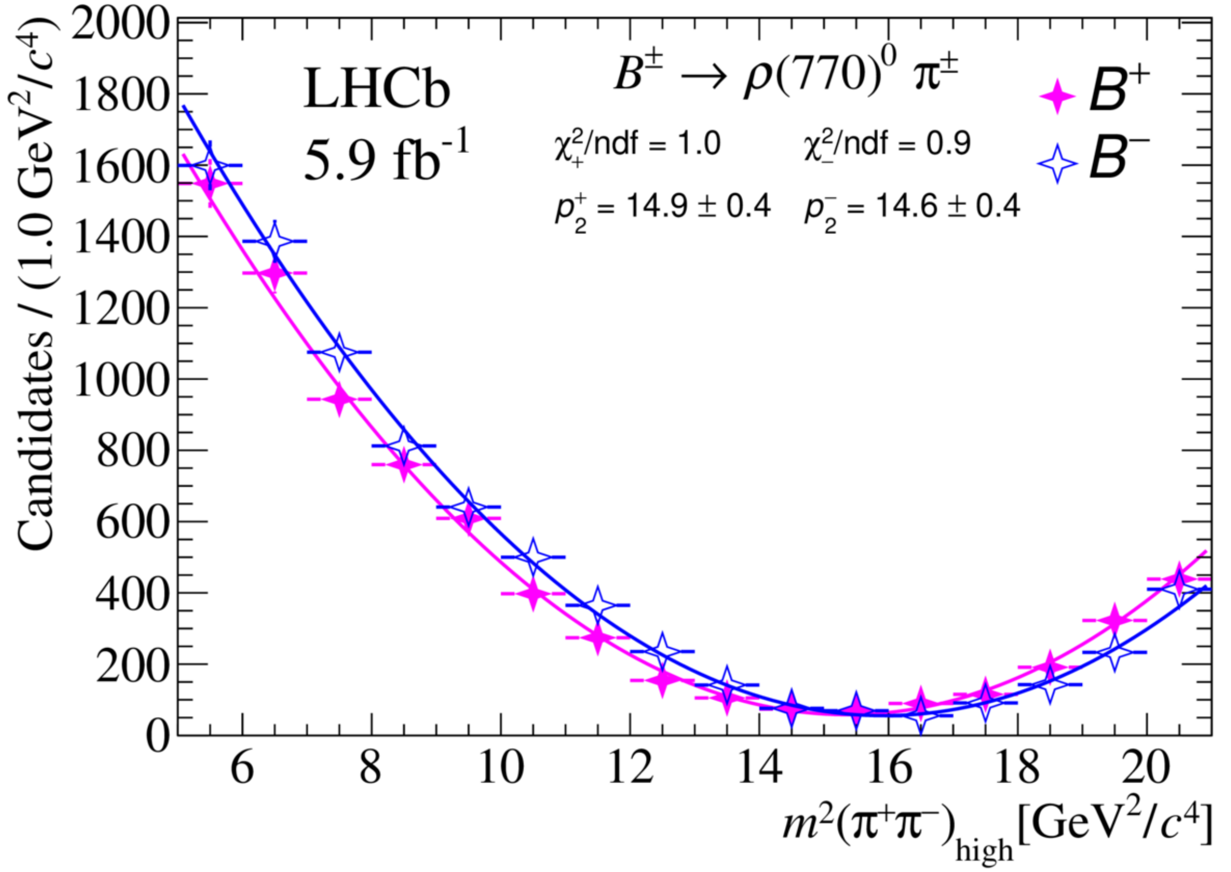
arXiv:2206.02038

- Background components formed of these resonances plus a random track: has an angular distribution similar to the scalar resonances, so it is absorbed in the $p^{\pm}_0$ parameter.

- In the π+π− P-wave, in the region dominated by the $B^{\pm} \rightarrow \rho(770)^0 K^{\pm}$ $A_{CP} = +0.150 \pm 0.019 \pm 0.011$
  - First observation of CP violation in this process

- Several other resonances tested: $B^{\pm} \rightarrow K^*(892)^0\pi^{\pm}$,B → $K^*(892)^0K^{\pm}$ and $B^{\pm} \rightarrow \phi(1020)K^{\pm}$ no other significant CP violation results





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MODEL DEPENDENT METHODS

Dalitz plot analysis features

- Interference plays a significant role in the phase space distributions and in the physics sensitivity
- Amplitude analysis can explore several features of multibody decays
  - Relative phases between states
  - Sensitivity to CP violating effects
  - Resolve ambiguities in weak phases
  - Hadron spectroscopy

Amplitude analysis

- Amplitude: sum of contributions
- $$\mathscr{A}(m_{12}^2, m_{23}^2) = \sum_{j=1}^{N} A_j(m_{12}^2, m_{23}^2) = \sum_{j=1}^{N} c_j F_j(m_{12}^2, m_{23}^2)$$

c_i : complex coefficients describing the relative magnitude and phase of the different isobars

F_i : dynamical amplitudes that contain the lineshape and spin-dependence of the hadronic part

Resonance mass term (e.g. Breit–Wigner)

Barrier factors - p, q: momenta of bachelor and resonance

Angular probability distribution

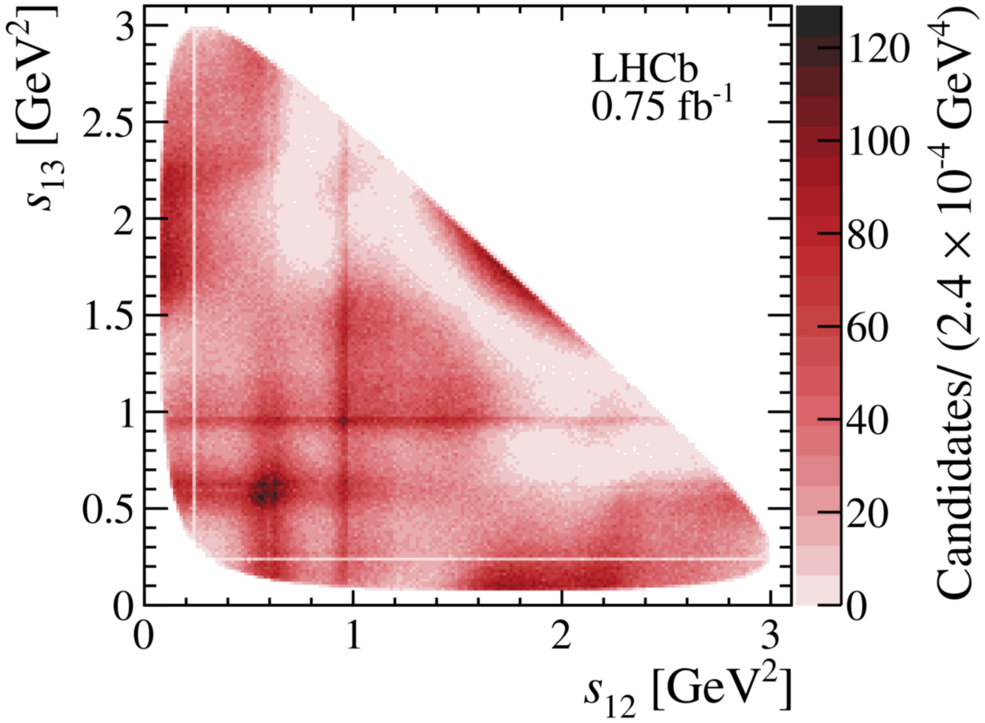
- S-wave (non-resonant component) description difficult, increasingly turning to multiple approaches
- Isobar: Each contribution has clear physical meaning
- K-matrix: Experimental interface scattering results that enforce 2-body unitarity
- Quasi-model-independent: Binned amplitude determined directly from data

S-wave dominant for $D^+_{(s)} \rightarrow \pi^-\pi^+\pi^+$

Quasi-model-independent partial- wave analysis, in which the $\pi^+\pi^-$ S-wave amplitude is parameterised as a generic complex function determined by a fit to the data.

$D^+ \rightarrow \pi^-\pi^+\pi^+$

ARXIV:2208.03300

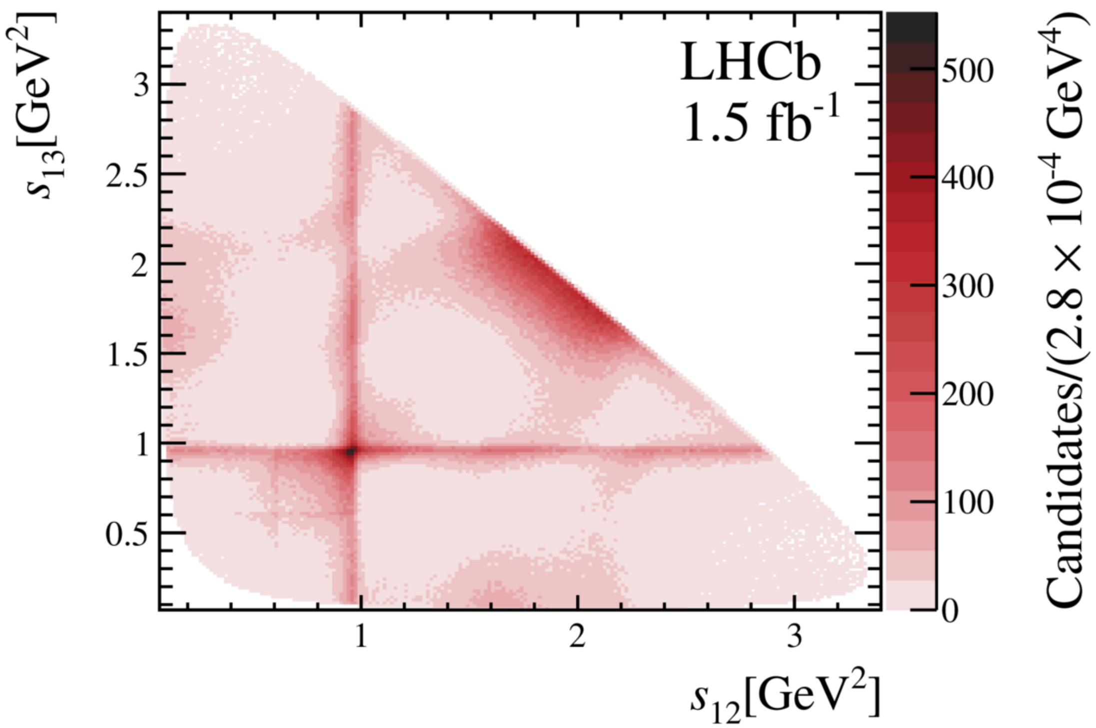


The S-wave component dominant, followed by the $\varrho(770)^0\pi^+$ and $f_2(1270)\pi^+$ components.

First observation of the $\omega(782) \rightarrow \pi^-\pi^+$ decay

$D^+_s \rightarrow \pi^-\pi^+\pi^+$

ARXIV:2209.09840



The S-wave dominant, followed by the contribution from spin-2 resonances and a small contribution from spin-1 resonances.

First observation of the $D_s^+ \rightarrow \omega(782)\pi^+$ channel in the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ decay.

*Also: PRD 106 (2022) 11, 112006.
PRD 79 (2009) 032003.*

S-wave dominant for $D^+_{(s)} \to \pi^-\pi^+\pi^+$

Quasi-mode

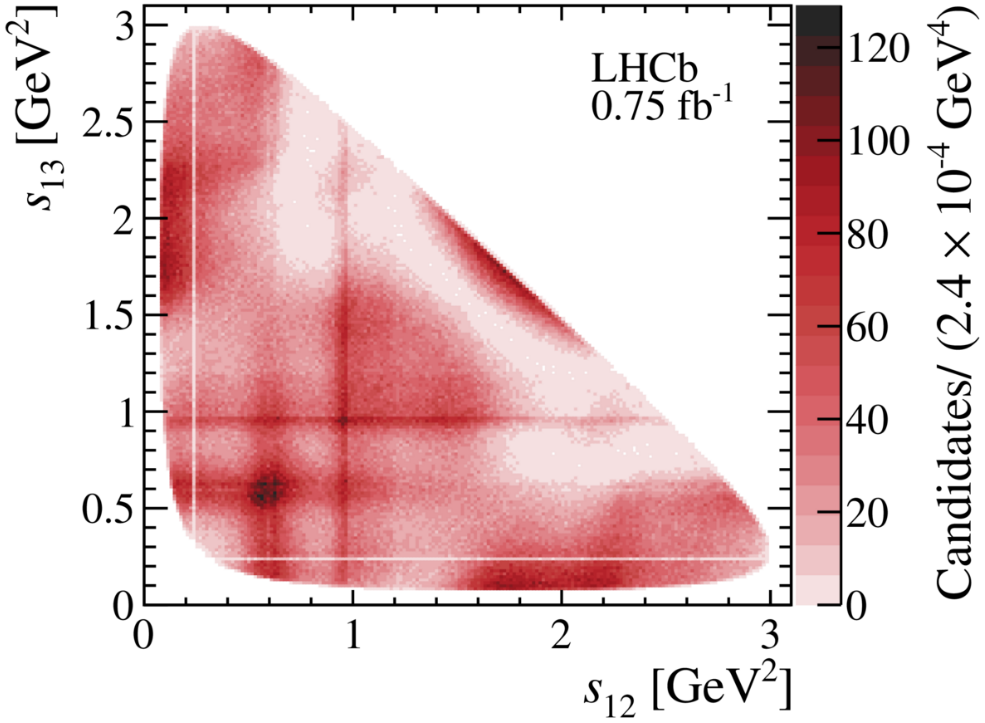
is paramet

$$\mathcal{A}(s_{12}, s_{13}) \equiv \mathcal{A}_S(s_{12}, s_{13}) + \sum_i a_i e^{i\delta_i} \mathcal{A}_i(s_{12}, s_{13}) + (s_{12} \leftrightarrow s_{13})$$

wave amplitude

it to the data.

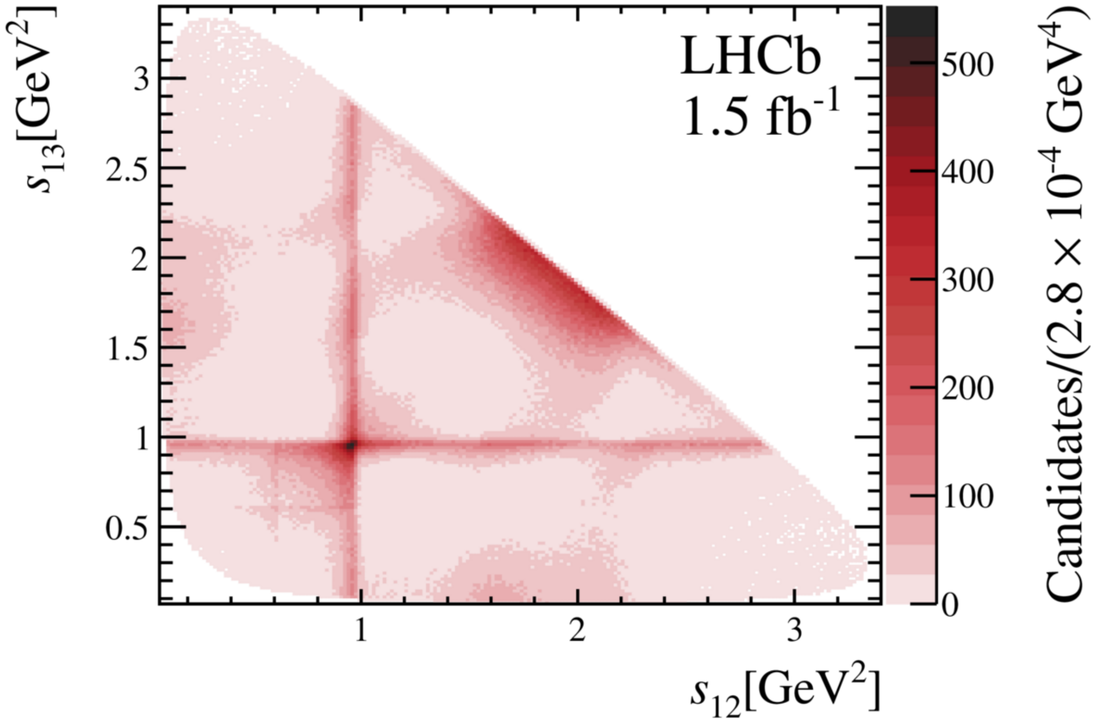
$D^+ \to \pi^-\pi^+\pi^+$ *ARXIV:2208.03300*



The S-wave component dominant, followed by the ϱ(770)$^0\pi^+$ and f2(1270)π^+ components.

First observation of the ω(782) → $\pi^-\pi^+$ decay

$D^+_s \to \pi^-\pi^+\pi^+$ *ARXIV:2209.09840*



The S-wave dominant, followed by the contribution from spin-2 resonances and a small contribution from spin-1 resonances.

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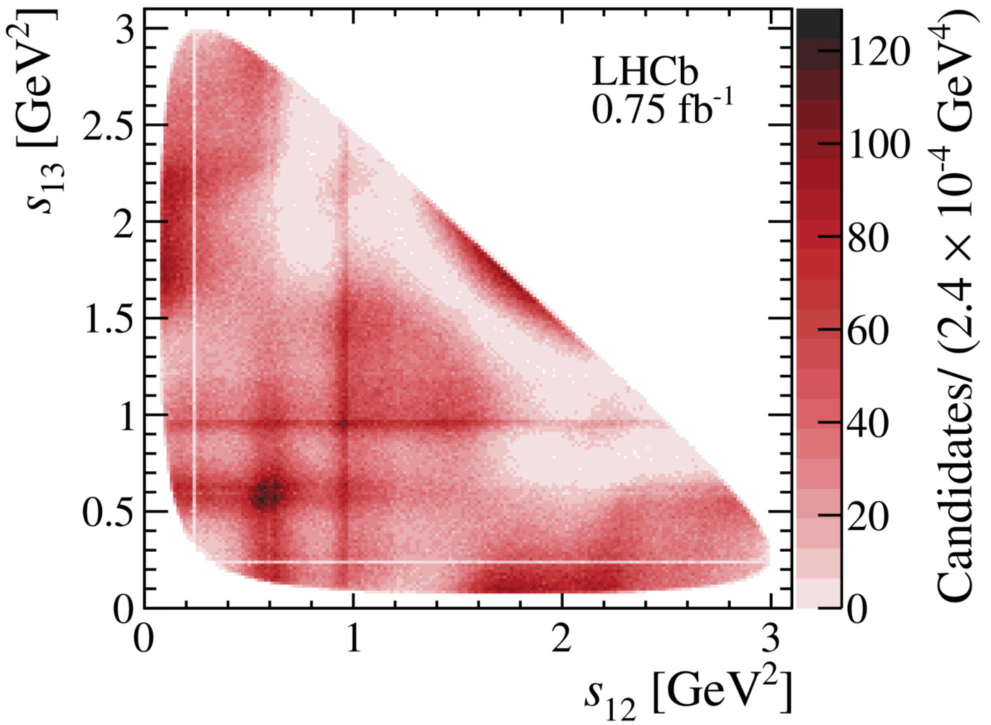
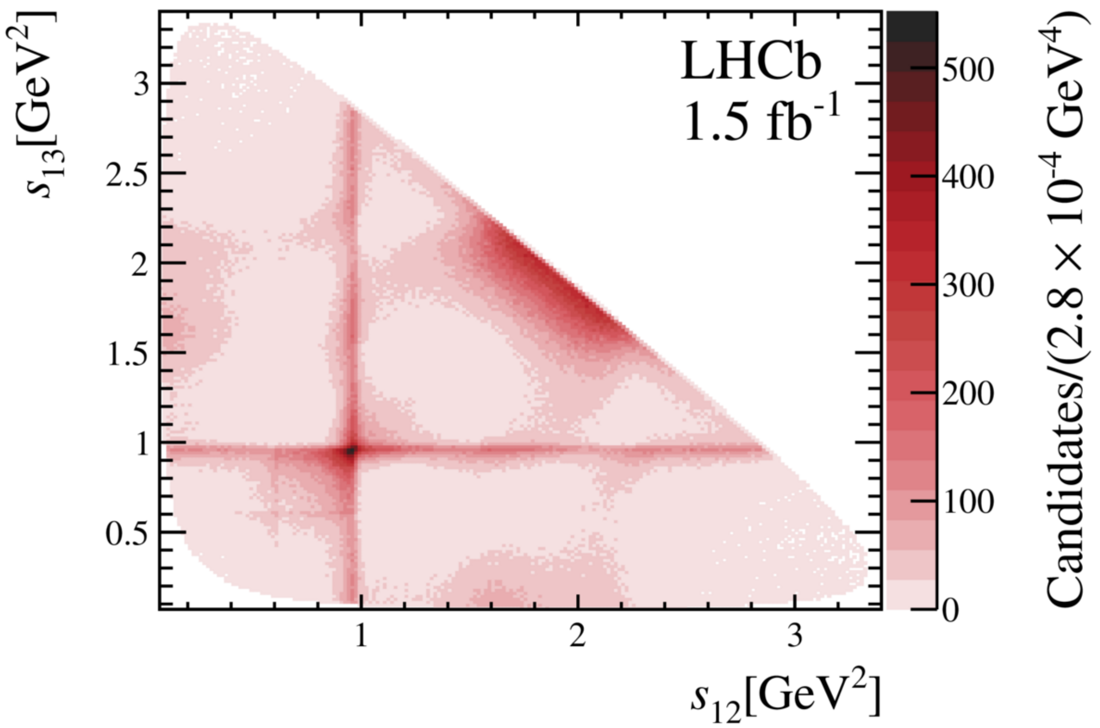
Also: PRD 106 (2022) 11, 112006.
PRD 79 (2009) 032003.

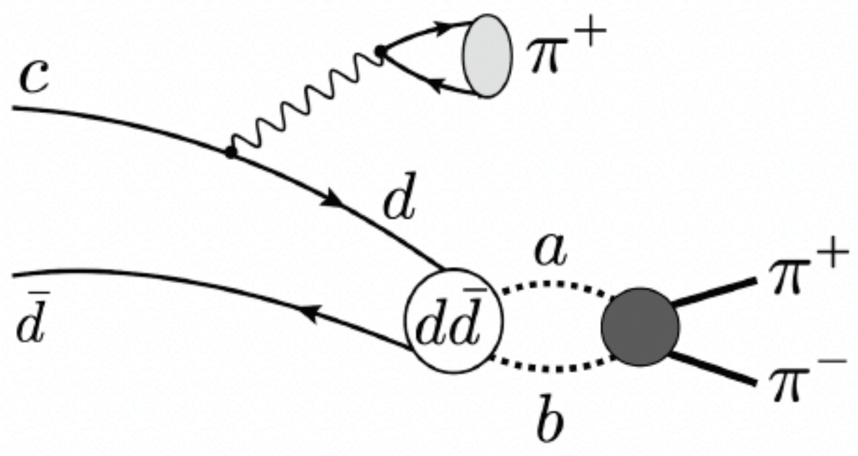
S-wave comparison

$D^+ \rightarrow \pi^-\pi^+\pi^+$

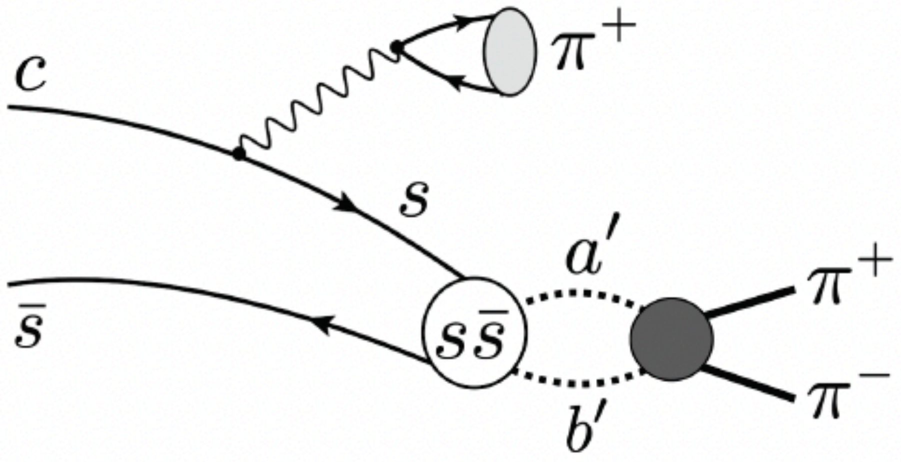


$D^+_s \rightarrow \pi^-\pi^+\pi^+$



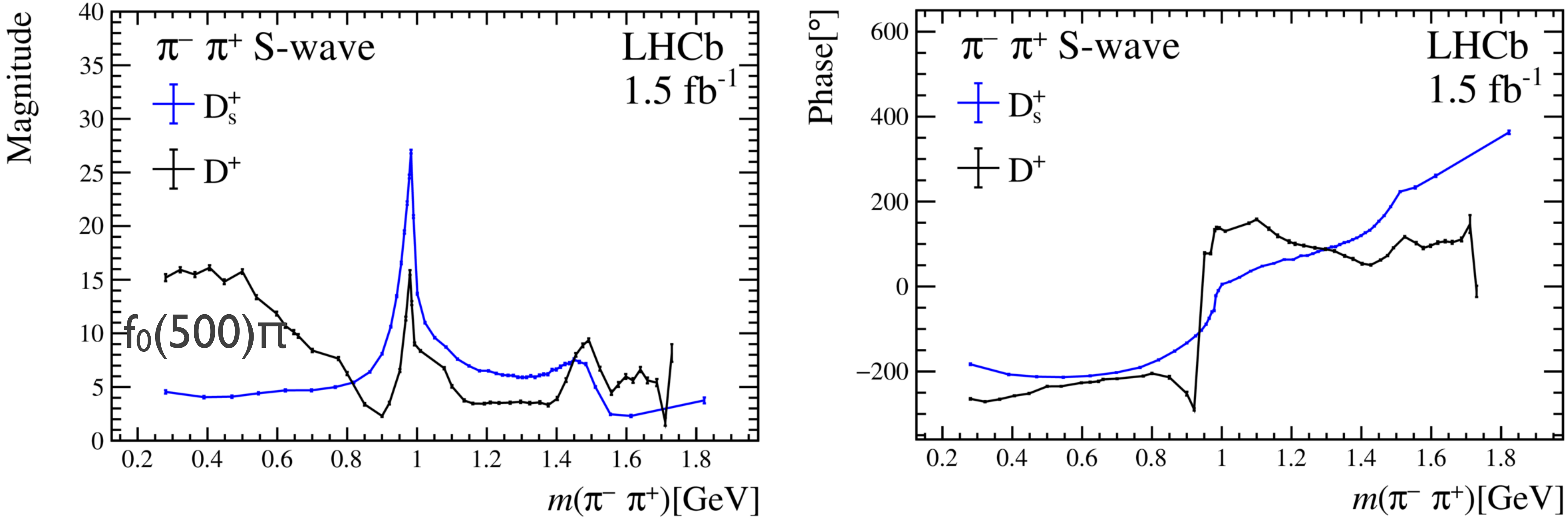


S-wave formed by the reactions $ab \rightarrow \pi\pi$,such as $\pi^+\pi^-$,$\pi^0\pi^0$ $\pi^0\eta$,$K^0\bar{K}^0$



S-wave formed by the reactions $ab \rightarrow \pi\pi$,with $s(\bar{u}u+\bar{d}d+\bar{s}s)\bar{s}$ such as K^+K^-,$K^0\bar{K}^0$,$\eta\eta$

S-wave comparison

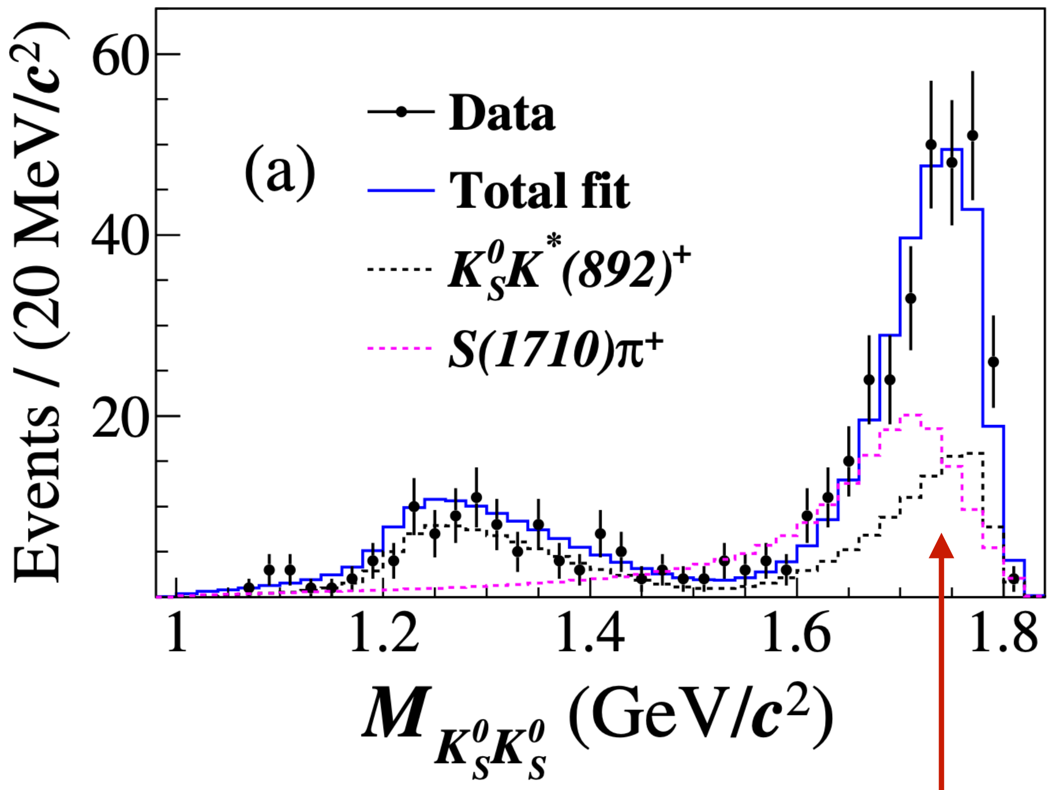


$f_0(980)\pi$: couples strongly to KK

mode	$D_s^+ \to \pi^-\pi^+\pi^+$	$D^+ \to \pi^-\pi^+\pi^+$
S-wave	84.97 ± 0.14	61.82 ± 0.5
P-wave	8.55 ± 0.44	32.31 ± 0.64
D-wave	13.12 ± 0.02	13.8 ± 0.2

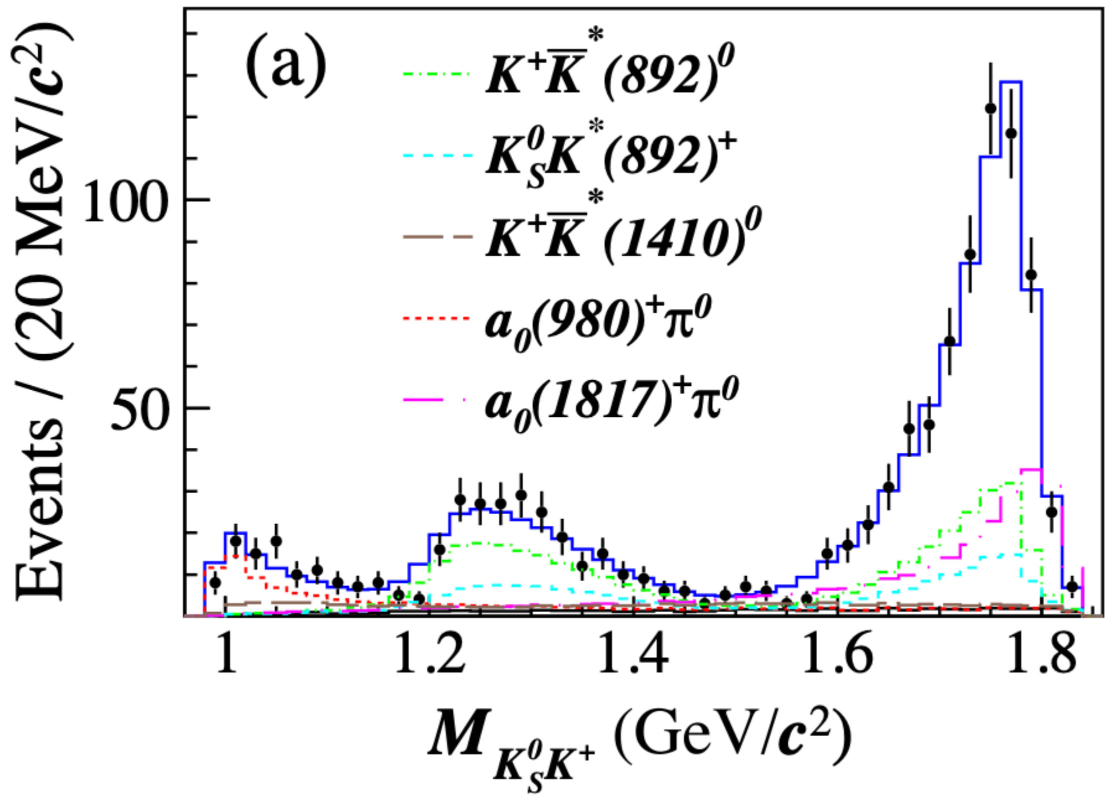
First amplitude analysis of $D^+_s \to K^0_S K\pi$

PHYS. REV. D 105, L051103 (2022)



$D^+_s \to K^0_S K^0_S \pi^+$

PRL129(2022)182001



$D^+_s \to K^0_S K^+ \pi^0$

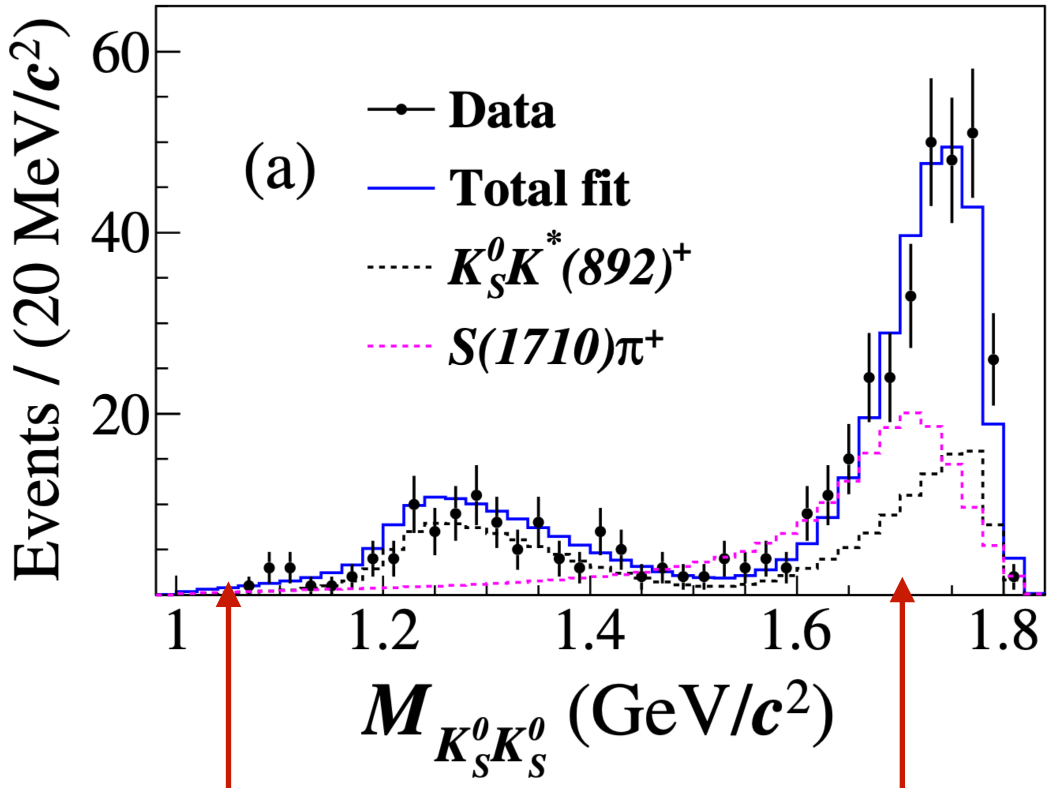
- 6.32 fb^{-1} collected at COM energies between 4.178 and 4.226 GeV
- Enhancement in $K^0_S K^0_S$ and $K^0_S K+$ near 1.7 GeV/c^2: isospin one partner of the $f_0(1710)$?
- Same resonance observed by BaBar in $\eta c \to \pi\pi\eta$?

PRD104(2021)072002

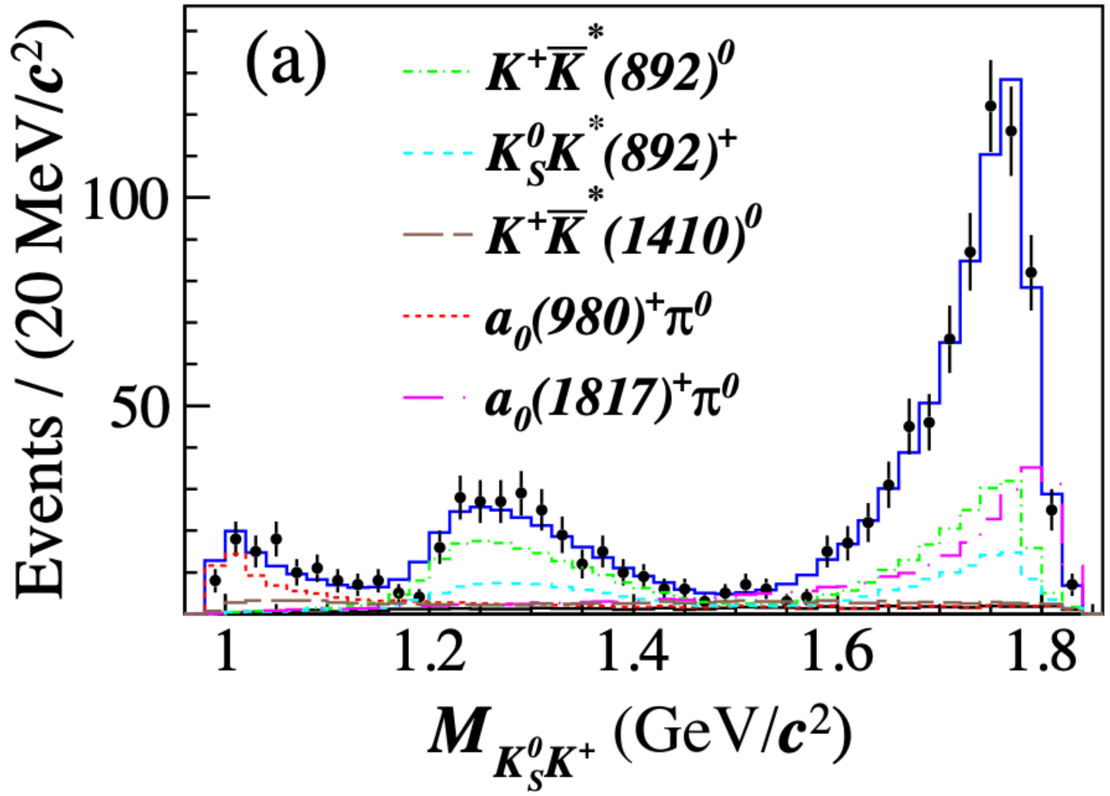
First amplitude analysis of $D^+_s \rightarrow K^0_S K \pi$

PHYS. REV. D 105, L051103 (2022)

PRL129(2022)182001



$D^+_s \rightarrow K^0_S K^0_S \pi^+$



$D^+_s \rightarrow K^0_S K^+ \pi^0$

destructive interference between $a_0(980)^0$ and $f_0(980)$

implies the existence of an isospin one partner of the $f_0(1710)$ meson, the $a_0(1710)^0$: supports KK* molecule hypothesis rather than a glue ball

$M = 1.817 \pm 0.008 \pm 0.020$ GeV/c^2,

$\Gamma = 0.097 \pm 0.022 \pm 0.015$ GeV/c^2

$\mathcal{B}(D_s^+ \rightarrow a_0(1817)^+\pi^0 = (3.44 \pm 0.52 \pm 0.32) \times 10^{-3}$ Significance > 10σ

THE ROYAL SOCIETY

BESIII

Loads of amplitude analyses

Amplitude analysis and branching fraction measurement of the decay D+ -> KS0 pi+ pi0 pi0
2305.15879

Amplitude analysis of D0 -> KL0 pi+ pi-
2212.09048

Amplitude analysis and branching fraction measurement of Ds+ -> K+pi+pi-pi0
2205.13759

Amplitude Analysis and Branching Fraction Measurement of Ds+ -> K+ pi+ pi-
2205.08844

Amplitude Analysis and Branching Fraction Measurement of Ds+ to KS K+ pi0
2204.09614

Amplitude Analysis and Branching Fraction Measurement of Ds to K+K-pi+pi+pi-
2203.06688

Amplitude Analysis and Branching Fraction Measurement of Ds+ -> pi+ pi0 eta'
2202.04232

Amplitude Analysis and Branching Fraction Measurement of Ds+ to Ks0Ks0pi+
2110.07650

Amplitude Analysis and Branching Fraction Measurement of Ds+ -> pi+pi0pi0
2109.12660

Dalitz-plot analysis of D_s^+ -> pi+pi-pi+
2108.10050

Amplitude analysis and branching fraction measurement of Ds+ -> eta pi+ pi+ pi-
2106.13536

Amplitude analysis of the decay D+ -> K+ Ks pi0
2104.09131

Amplitude Analysis and Branching Fraction Measurement of Ds+ -> Ks pi+ pi0
2103.15098

Amplitude Analysis and Branching Fraction Measurement of Ds+ -> K-K+pi+pi0
2103.02482

Amplitude Analysis and Branching Fraction Measurement of Ds+ -> K-K+pi+pi0
2103.02482

Amplitude Analysis and Branching Fraction Measurement of Ds ->K+K-pi+
2011.08041

Amplitude Analysis and BF measurement of D0 -> K- pi+ 2pi0
1903.06316

Amplitude analysis of Ds+ -> pi+pi0eta
1903.04118

Amplitude analysis of D+ -> KS pi+ pi+ pi-
1901.05936

Amplitude analysis of D0 to K-pi+pi+pi-
1701.08591

BESIII: PRD 95 (2017) 7, 072010

Dalitz analysis of $D^0 \to K^-\pi^+\eta$ decays at Belle
Belle Collaboration, published in PRD 102, 012002 (2020 July 6

D to $4h^{(\prime)}$ decays : LHCb: JHEP 02 (2019) 126
LHCb: Eur. Phys. J. C78 (2018) 443
CLEO-c data: JHEP 05 (2017)

Summary

- Unique sensitivity to CP violation
- Important for QCP tests
- A plethora of methods to explore multibody decays
- Development of model unbiased methods is an interesting challenge
- Many interesting results, very active field