Charm in Beauty

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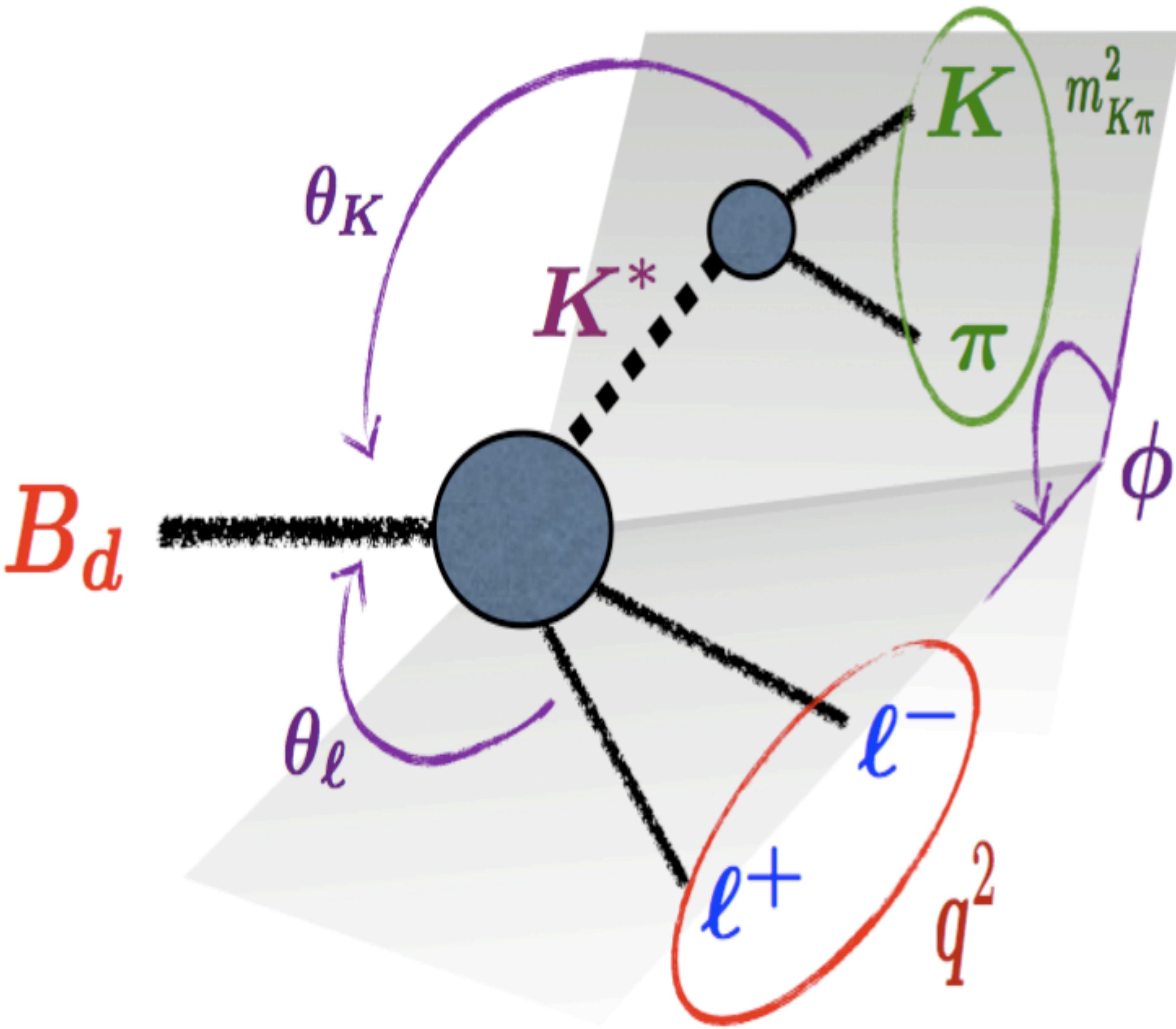
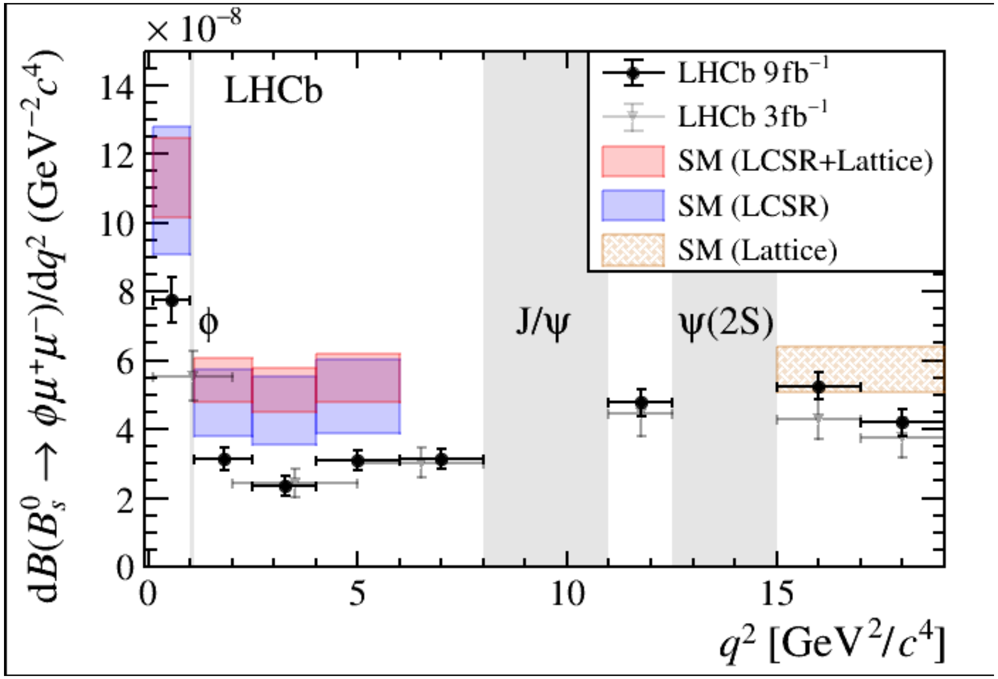
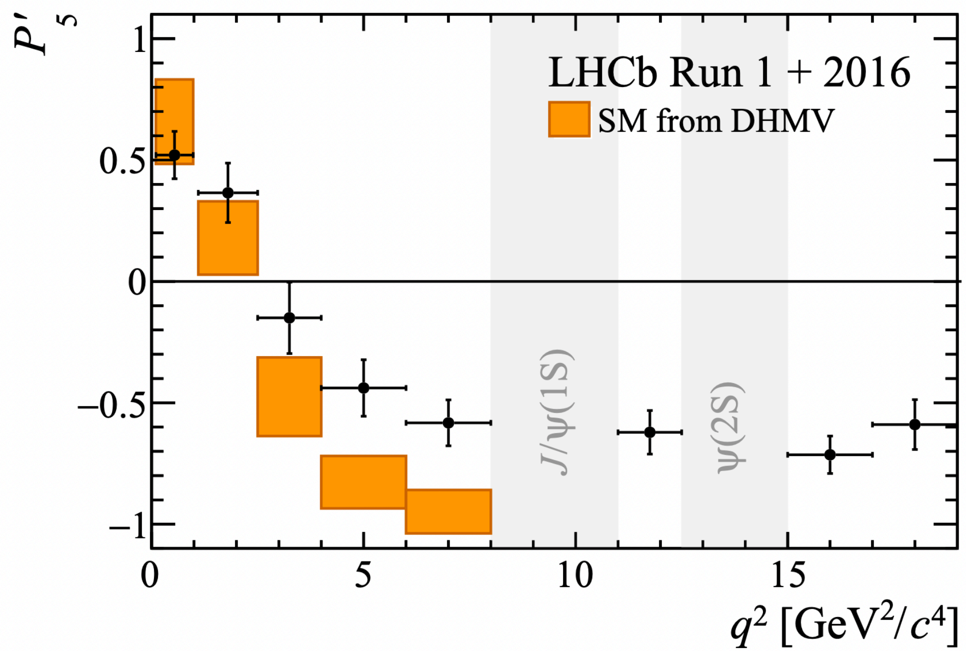
Heavy Flavour 2023 Quo Vadis? Islay June 2023


θ_K
K^*
K
π
$m^2_{K\pi}$
ϕ
B_d
θ_ℓ
ℓ^-
ℓ^+
q^2

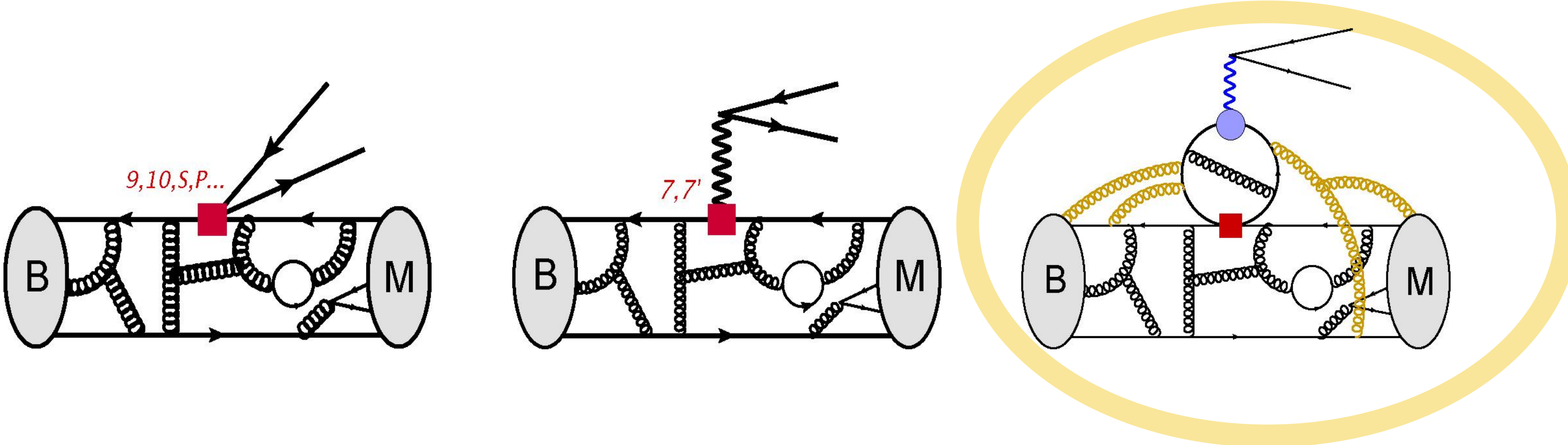


Branching fractions

Angular observables



Anatomy of $B \to M_\lambda \ell^+ \ell^-$ EFT Amplitudes

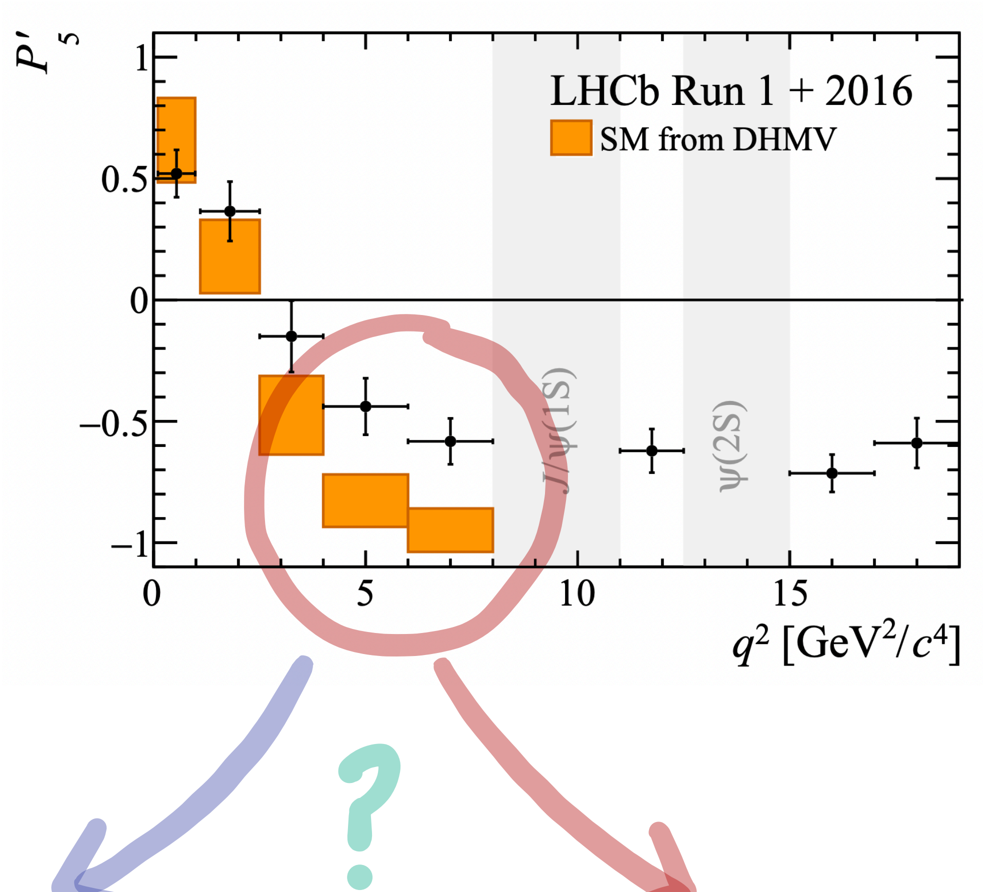


$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\} + \mathcal{O}(\alpha^2)$$

1   ~ 0.2

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \bar{s}\, \Gamma_\lambda^{(T)}\, b\, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i\, \mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{ j_{\rm em}^\mu(x), \mathcal{C}_i\, \mathcal{O}_i(0)\} | \bar{B}(q+k) \rangle$

The BSM/QCD dichotomy



$C_i \neq C_i^{SM}$ (BSM)

$\langle \mathcal{O}_i \rangle \neq \langle \mathcal{O}_i \rangle^{\text{calculated}}$ (QCD)

Here we are discussing the problem of calculating

- Local Form Factors :

$$\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) |\, \bar{s}\, \Gamma_\lambda^{(T)}\, b\, | \bar{B}(k+q) \rangle$$

- Non-Local Form Factors:

$$\mathcal{H}^\mu(q^2) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{ j^\mu_{\rm em}(x), [\bar{b}_L \gamma^\nu c_L][\bar{c}_L \gamma_\nu s](0) \} | \bar{B}(q+k) \rangle$$

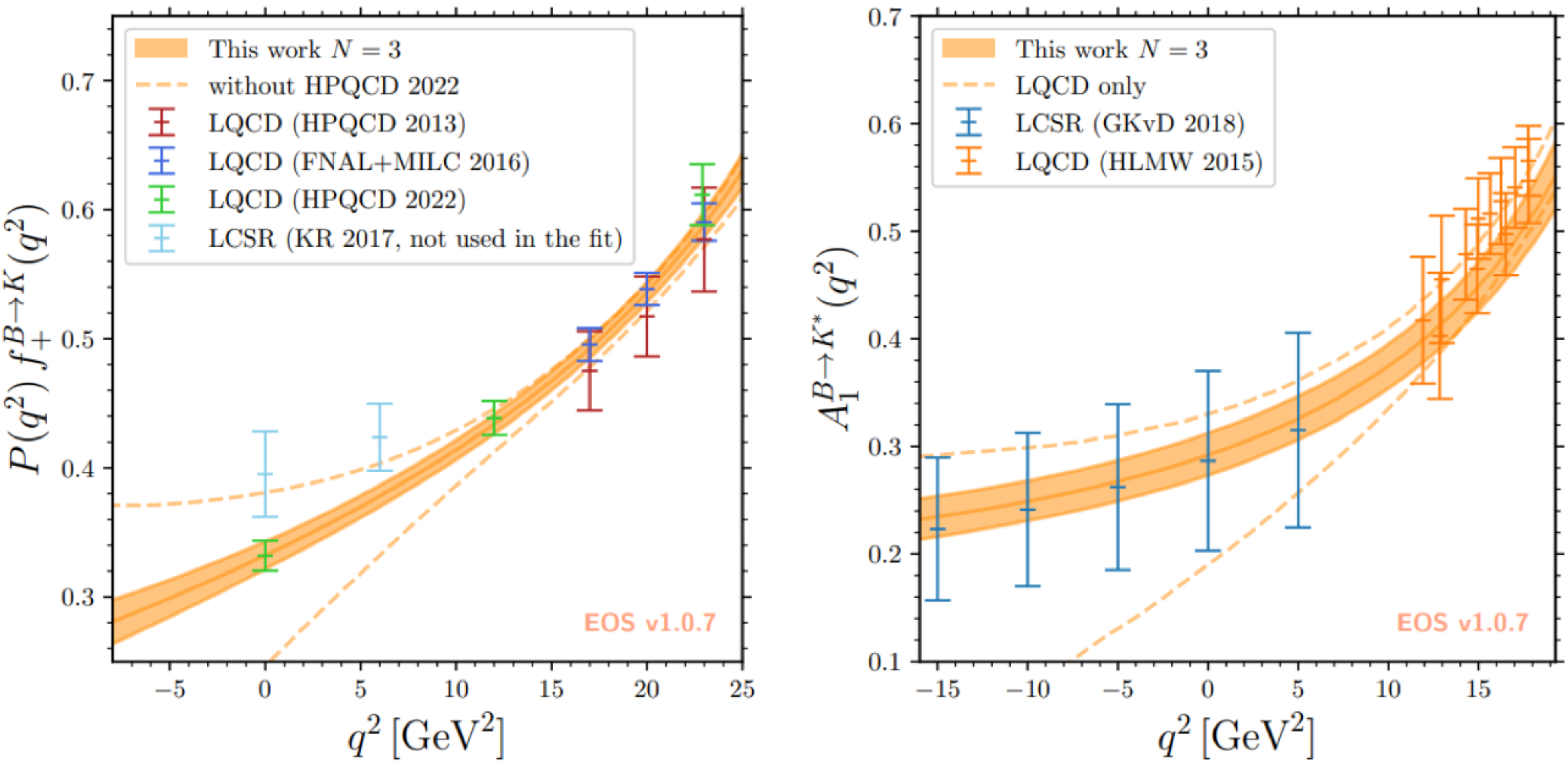
In **both cases** the *“modern” strategy* is

- **Calculate/extract** the form factors in optimal/feasible kinematic regions (**not** necessarily physical or the regions we are interested in)
  $\longrightarrow$ **“data”** (LQCD, LCSRs, exp. data, ...)

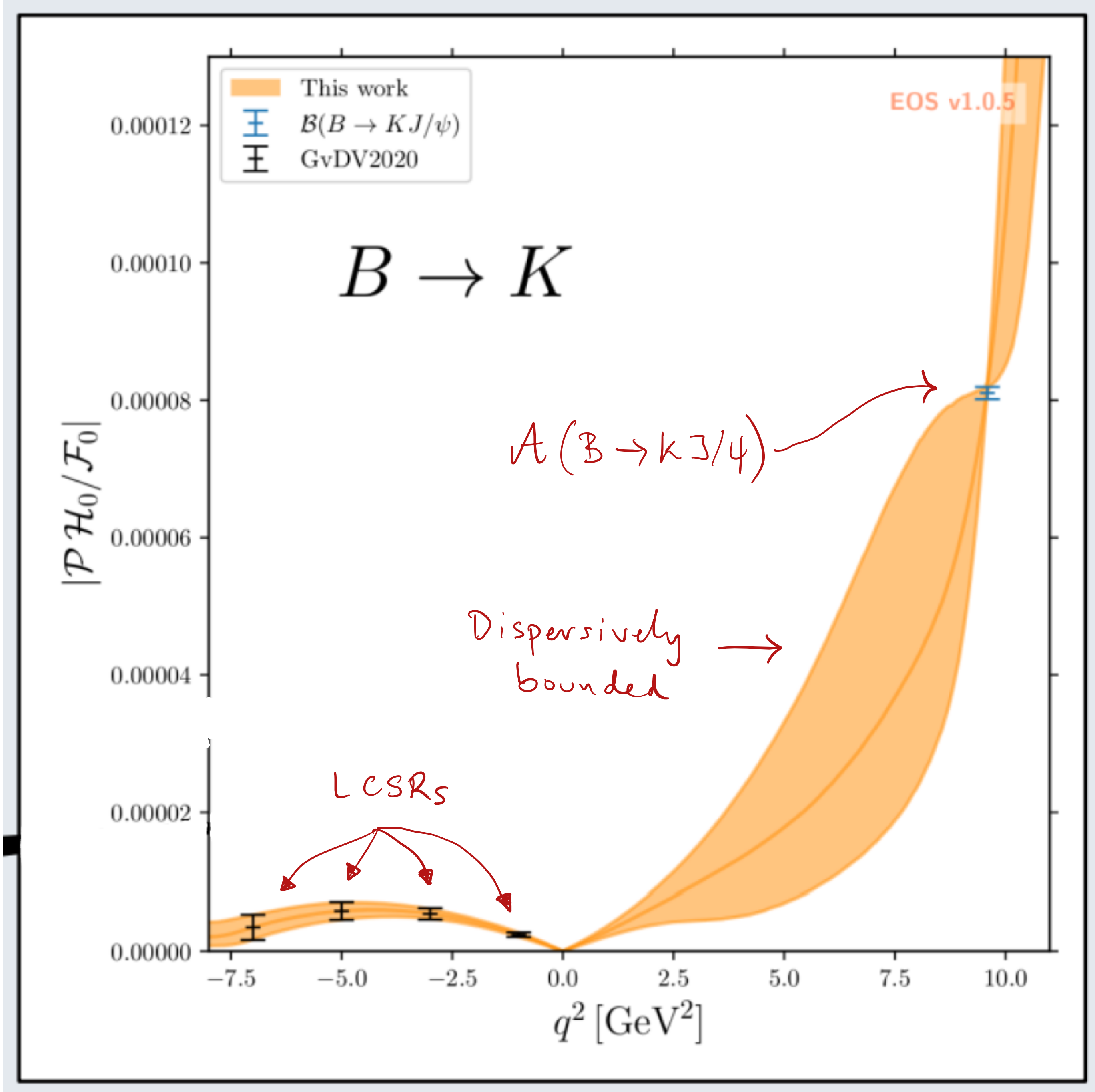
- **Parametrize** q^2 dependence by means of a rigorous **analytic expansion**

- **Fit** the (truncated) parametrization to the **“data”**

- Control the truncation error by means of a **dispersive bound**.



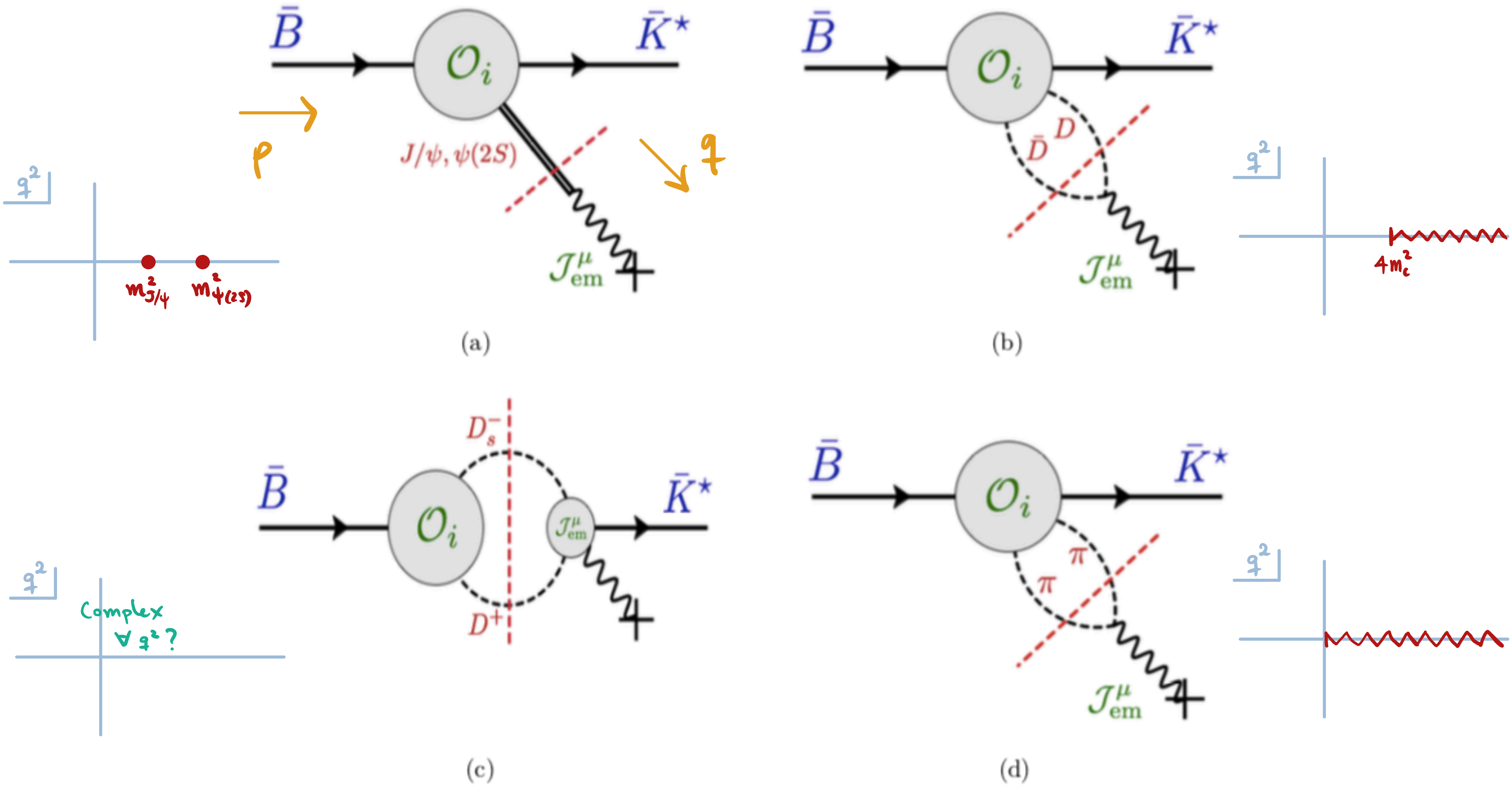
This work $N = 3$
without HPQCD 2022
LQCD (HPQCD 2013)
LQCD (FNAL+MILC 2016)
LQCD (HPQCD 2022)
LCSR (KR 2017, not used in the fit)
EOS v1.0.7
$P(q^2)\ f_+^{B\to K}(q^2)$
q^2 [GeV^2]
This work $N = 3$
LQCD only
LCSR (GKvD 2018)
LQCD (HLMW 2015)
EOS v1.0.7
$A_1^{B\to K^*}(q^2)$
q^2 [GeV^2]



This work
$\mathcal{B}(B \to K J/\psi)$
GvDV2020
EOS v1.0.5
$B \to K$
A (B → K J/ψ)
Dispersively bounded
LCSRs
$|\mathcal{P}\,\mathcal{H}_0/\mathcal{F}_0|$
0.00012
0.00010
0.00008
0.00006
0.00004
0.00002
0.00000
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
q^2 [GeV^2]

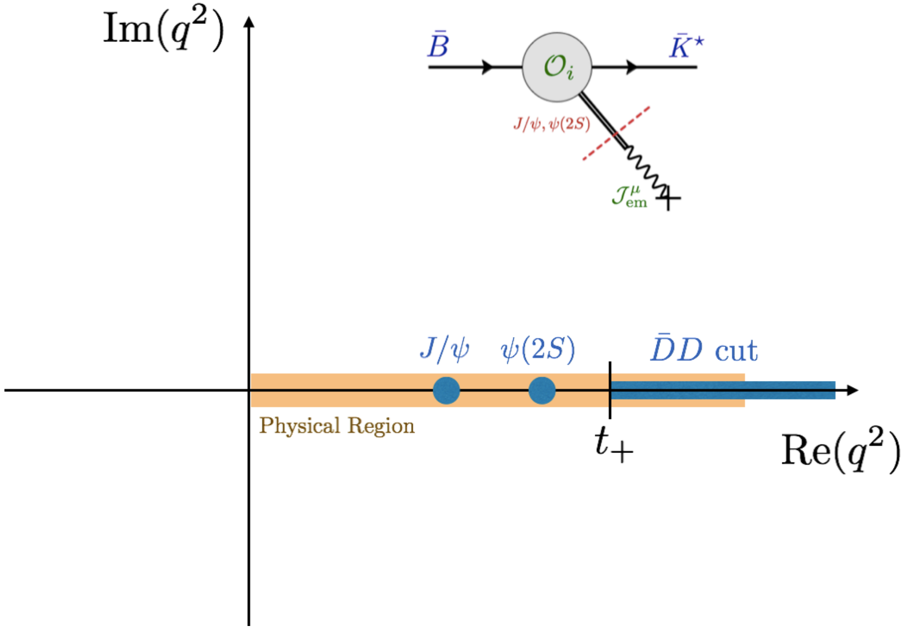
Parametrization of q2 dependence

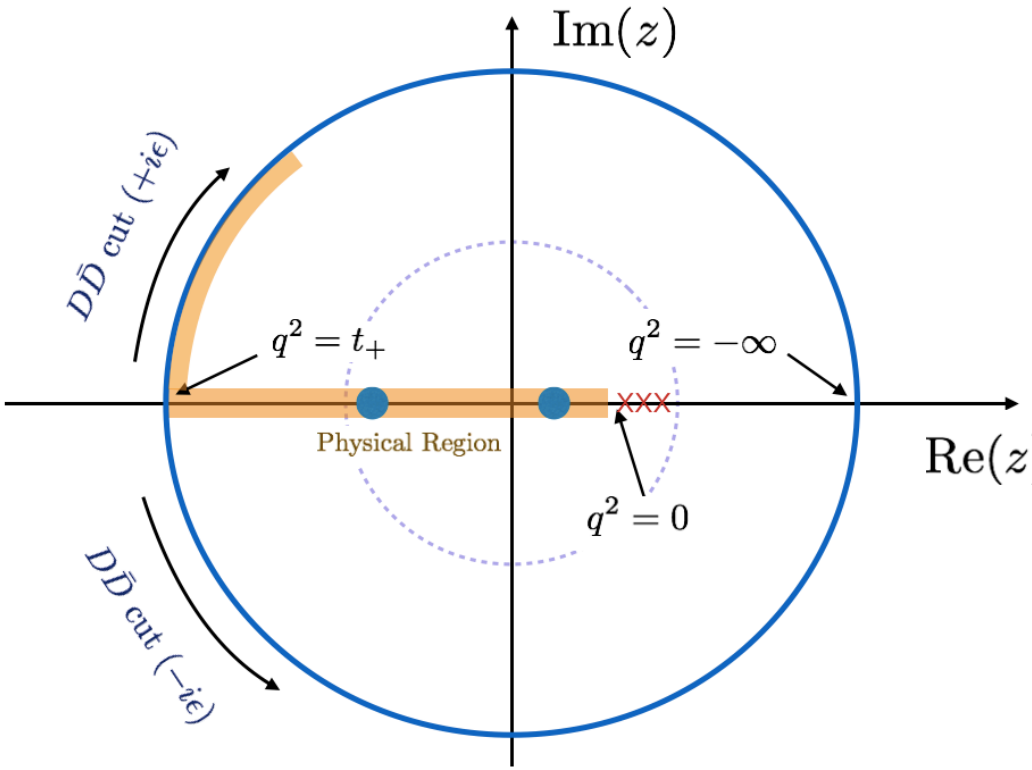
Non-Local Form Factors: Analytic structure



Analytic continuation to physical q^2

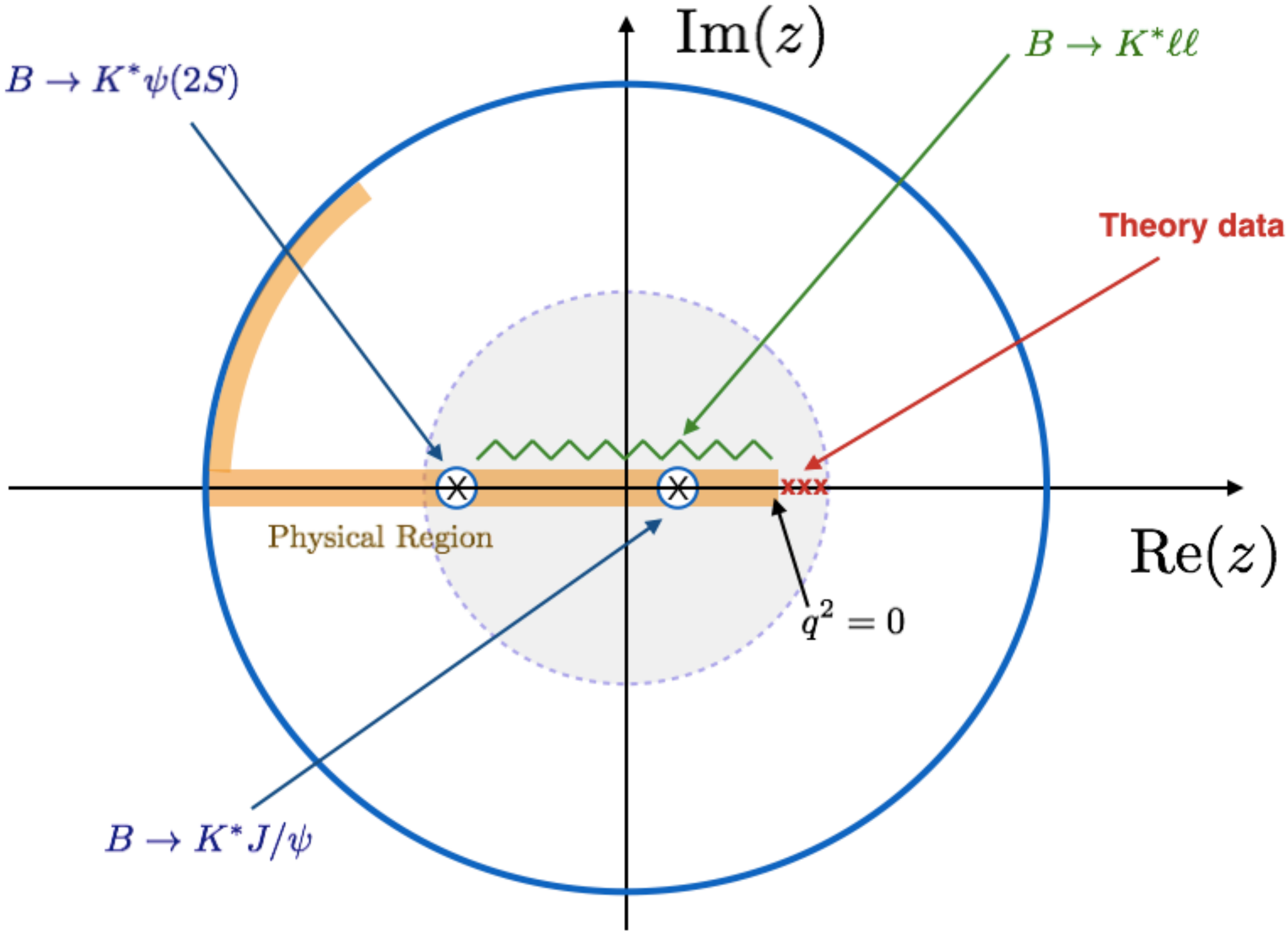
z-parametrisation for $\mathcal{H}_\lambda(q^2)$





- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$ is **analytic in** $|z| < 1$
- Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$: $$\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{H}_\lambda(z)$$
- Expansion needed for $|z| < 0.52$ $\left(-7\,\mathrm{GeV}^2 \leq q^2 \leq 14\mathrm{GeV}^2\right)$

Fit to z-parametrisation



Experimental constraints :

- The residues of the poles are given by $B \to K^* \psi_n$:

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \cdots$$

- Angular analyses Belle, Babar, LHCb determine :

$$|r_\perp^{\psi_n}|,\ |r_{||}^{\psi_n}|,\ |r_0^{\psi_n}|,\ \arg\{r_\perp^{\psi_n} r_0^{\psi_n *}\},\ \arg\{r_{||}^{\psi_n} r_0^{\psi_n *}\},$$

where $\quad r_\lambda^{\psi_n} \equiv \underset{q^2 \to M_{\psi_n}^2}{\text{Res}} \dfrac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \dfrac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2\, \mathcal{F}_\lambda(M_{\psi_n}^2)}$

Calculations in specific regions

Non-local form factors: Operator Product Expansion

$$\mathcal{H}^\mu(q,k) = i\int d^4x\, e^{iq\cdot x}\, \langle \bar{M}_\lambda(k)|\mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\,\mathcal{O}_i(0)\}|\bar{B}(q+k)\rangle$$

- Large-q^2: Dominated by $x \sim 0$ (short-distance dominance - OPE)

  Grinstein, Pirjol; Beylich, Buchalla, Feldmann

- Low-q^2: Dominated by $x^2 \sim 0$ (light-cone dominance - LCOPE)

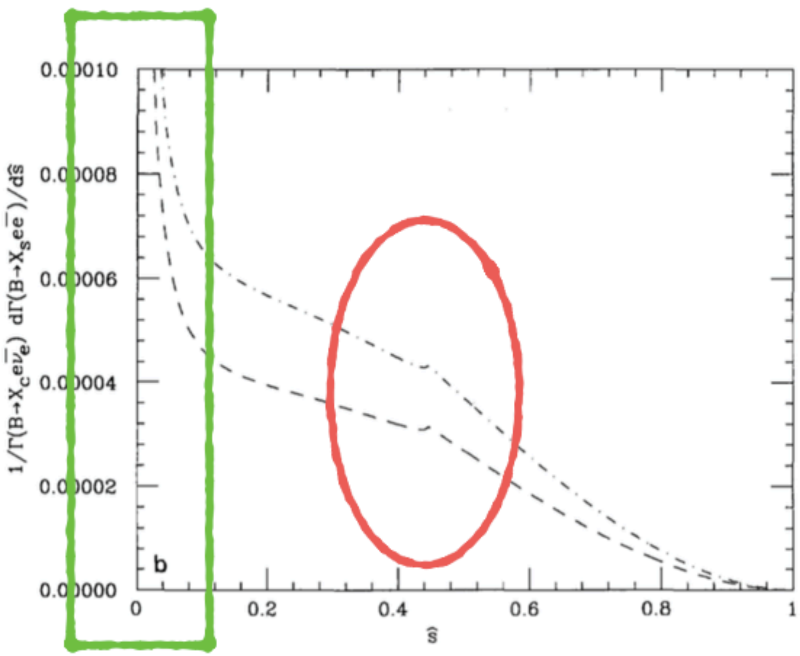
  Khodjamirian, Mannel, Pivovarov, Wang

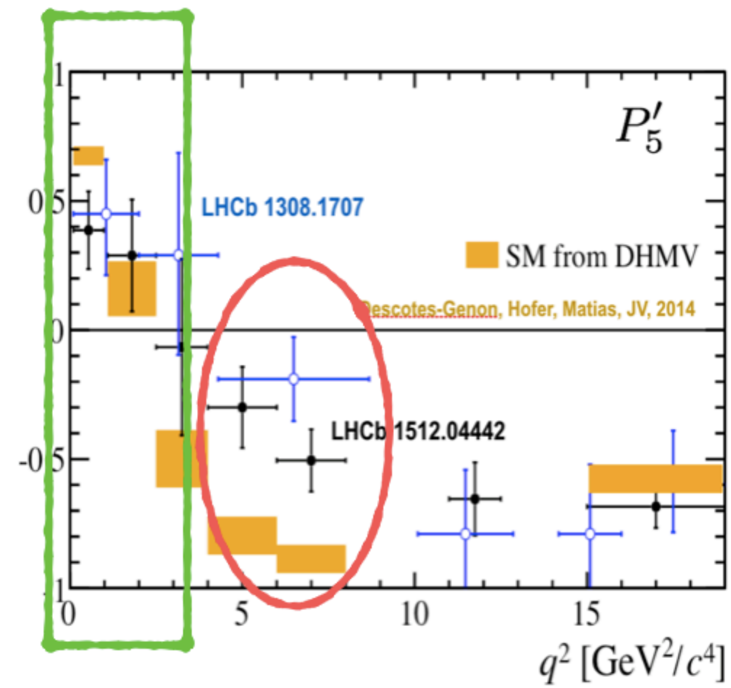
Non-local form factors: Importance of on-shell cuts

- QCD Factorization   Beneke, Feldmann, Seidel 2001

$$\mathcal{H}_\lambda(q^2) \sim \Delta C_9^\lambda(q^2)\,\mathcal{F}_\lambda(q^2) + \frac{1}{q^2}\Delta C_7^\lambda(q^2)\,\mathcal{F}_\lambda^T(q^2) + HSS + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$

- It is assumed that the charm loop is dominated by short distances





- Kink at $q^2 = 4m_c^2$ symptom of breaking of perturbativity

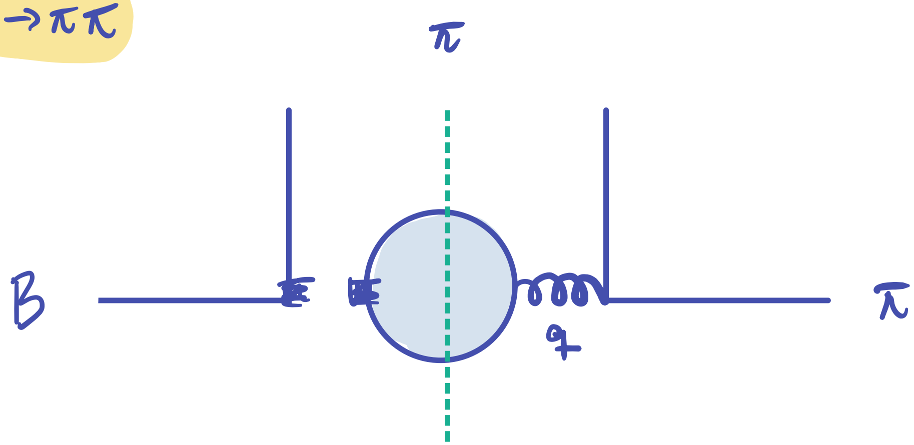
Non-local form factors: QCDF

What is QCD Factorization doing for us?

Beneke, Feldmann, Seidel 2001

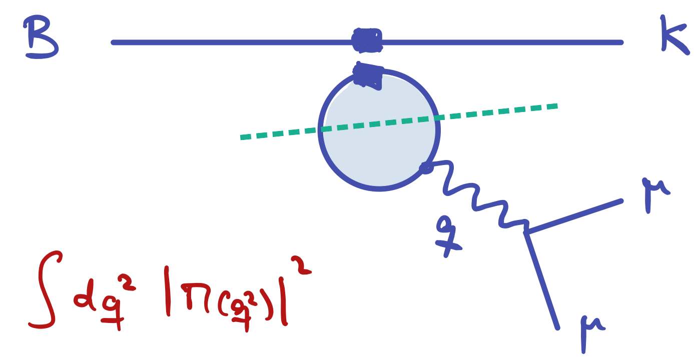
Beneke, Buchalla, Neubert, Sachrajda 2009

$B \to \pi\pi$



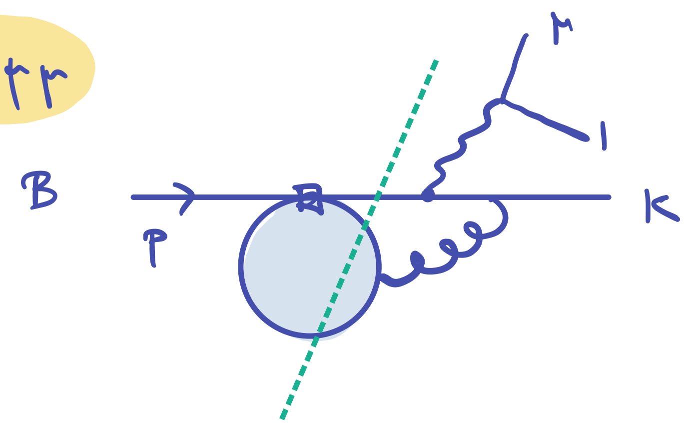
$$\left| \int dq^2\, \Pi(q^2) \right|^2$$

$B \to K\mu\mu$



$$\int dq^2\, |\Pi(q^2)|^2$$

$B \to K\mu\mu$



← This imaginary part is like the one in $B \to \pi\pi$.

(See also: Beylich, Buchalla, Feldmann.)

Non-local form factors: Operator Product Expansion

We write

$$\mathcal{H}^\mu(q,k) = \langle \bar{M}_\lambda(k)|\mathcal{K}^\mu(q)|\bar{B}(q+k)\rangle$$

With the operator $\mathcal{K}^\mu(q)$ given by

$$\mathcal{K}^\mu(q) = i\int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\, \mathcal{O}_i(0)\}$$

It turns out that: **Leading-order OPE = Leading order LCOPE**

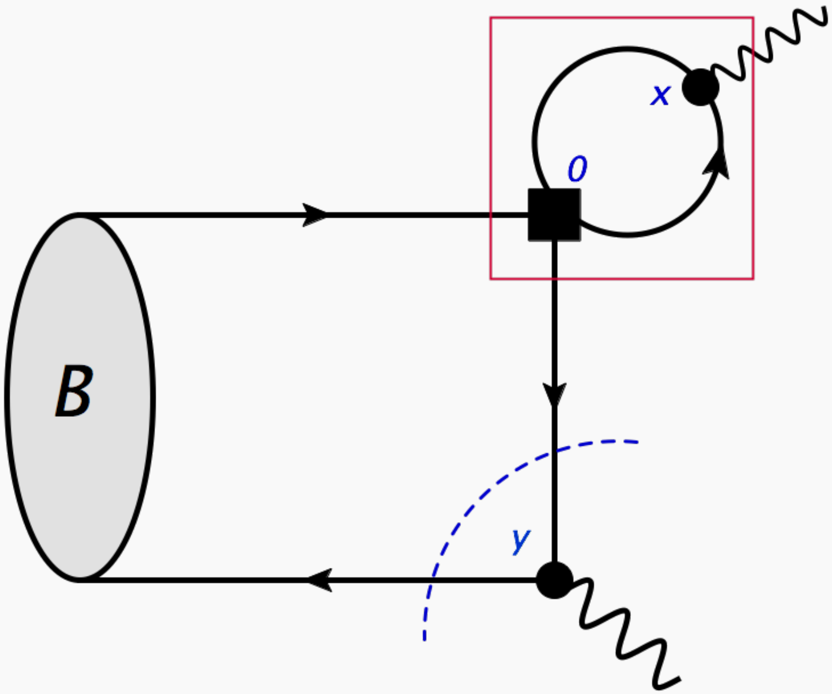
$$\mathcal{K}^\mu_{\rm OPE}(q) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\,\bar{s}\gamma_\nu P_L b + \Delta C_7(q^2)\, 2im_b\, \bar{s}\sigma^{\mu\nu} q_\nu P_R b + \cdots$$

With this we have:

$$\mathcal{H}^\mu_{\rm OPE}(q,k) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\mathcal{F}_\nu + 2im_b\, \Delta C_7(q^2)\mathcal{F}^{T\mu} + \cdots$$

LCOPE very low q^2

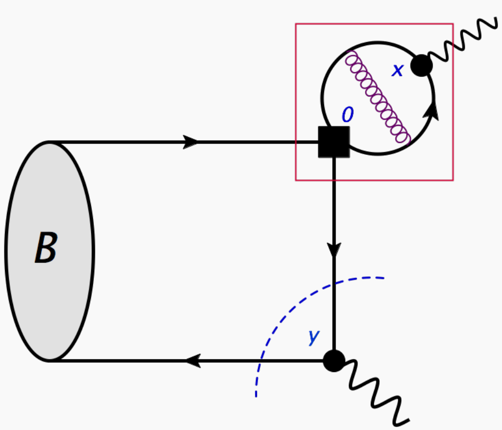
- LCSRs with B-meson DAs   Khodjamirian, Mannel, Pivovarov, Wang

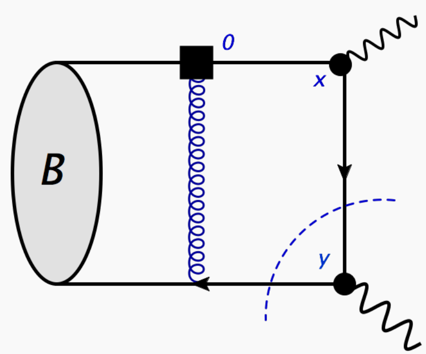


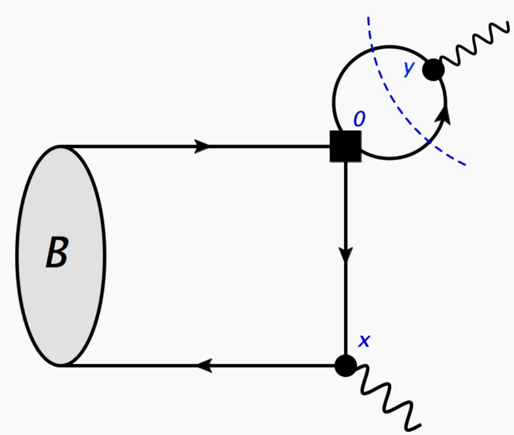
LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$\Rightarrow \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{LCSR}$$

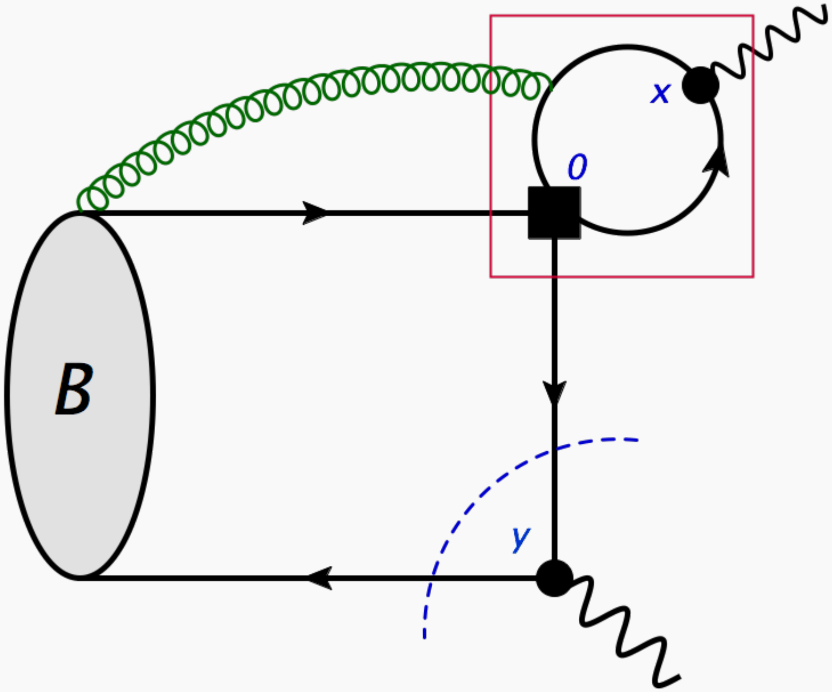






LCOPE at very low q^2 – Subleading power

- LCSRs with B-meson DAs   Khodjamirian, Mannel, Pivovarov, Wang

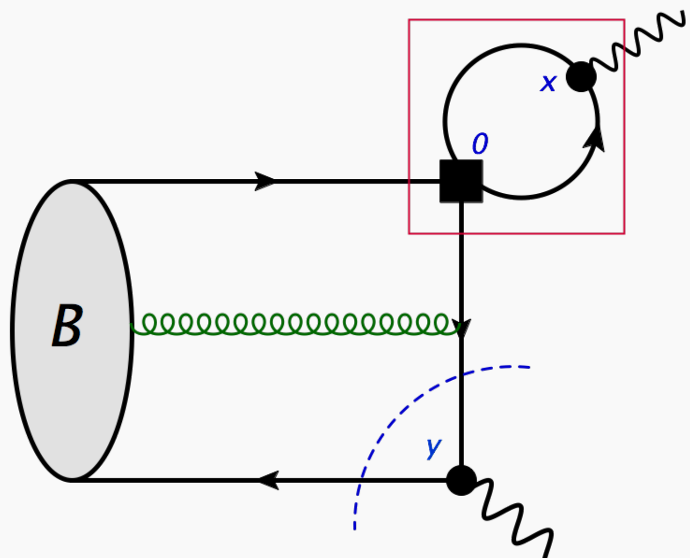


LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] +$$

$$+\ (\text{coeff}) \times [\,\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,]\ + \cdots$$

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$



LCOPE at very low q^2 – Subleading power

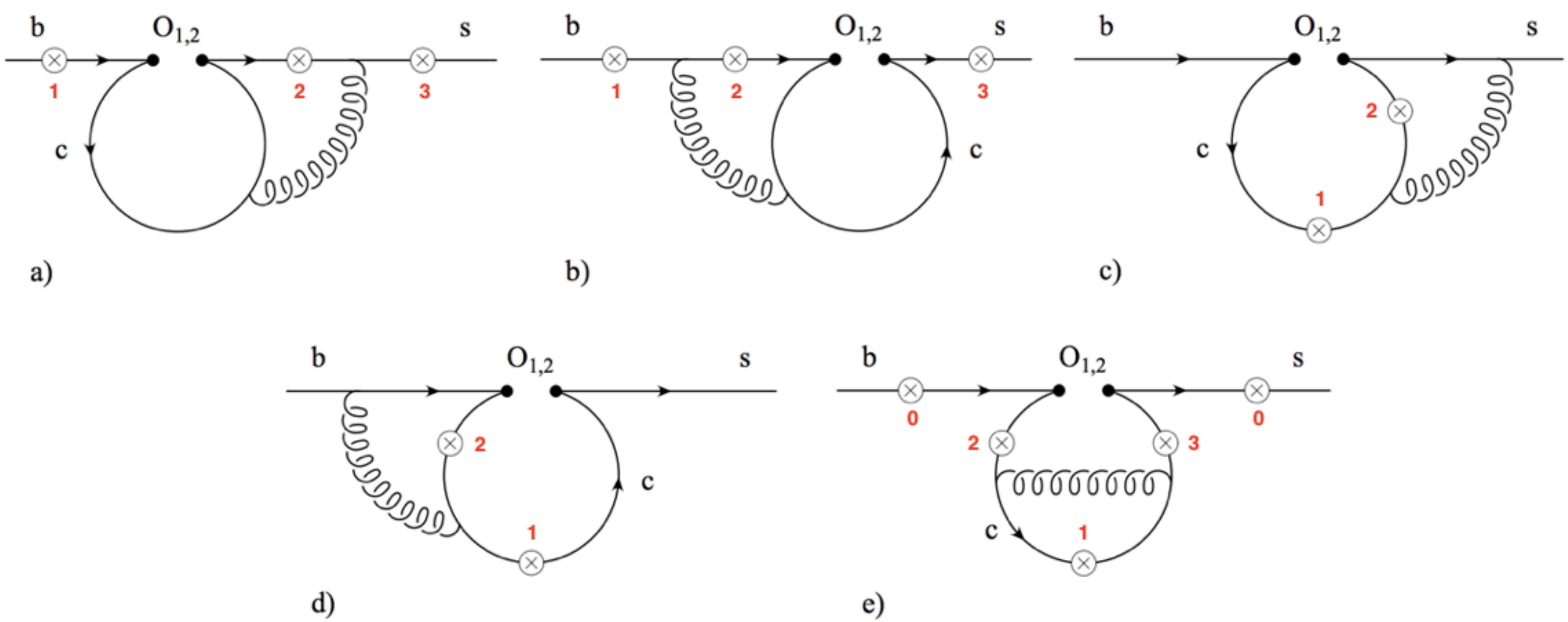
Recalculation of charm-loop effect Gubernari, van Dyk, Virto, 2011.09813

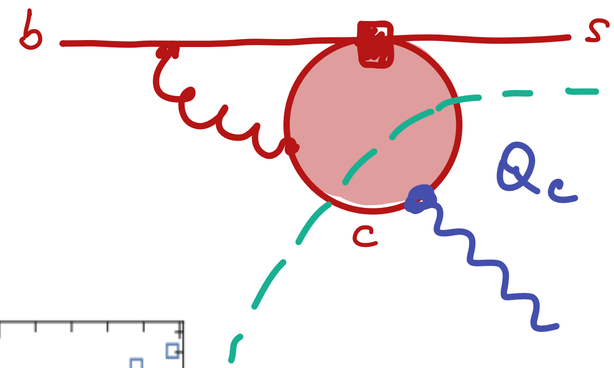
Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\text{GeV}^2)$	This work	Ref. [11]
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\text{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\text{GeV}$
$B \to K^*$	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\text{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\text{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\text{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\text{GeV}$
	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\text{GeV}$	—
$B_s \to \phi$	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\text{GeV}$	—

- We reproduce the result of KMPW'2010
- We incude complete set of 3-particle LCDAs Braun,Li,Manashov 2017
- Cancellations + Parametric lead to a reduction of the effect of **two orders of magnitude**

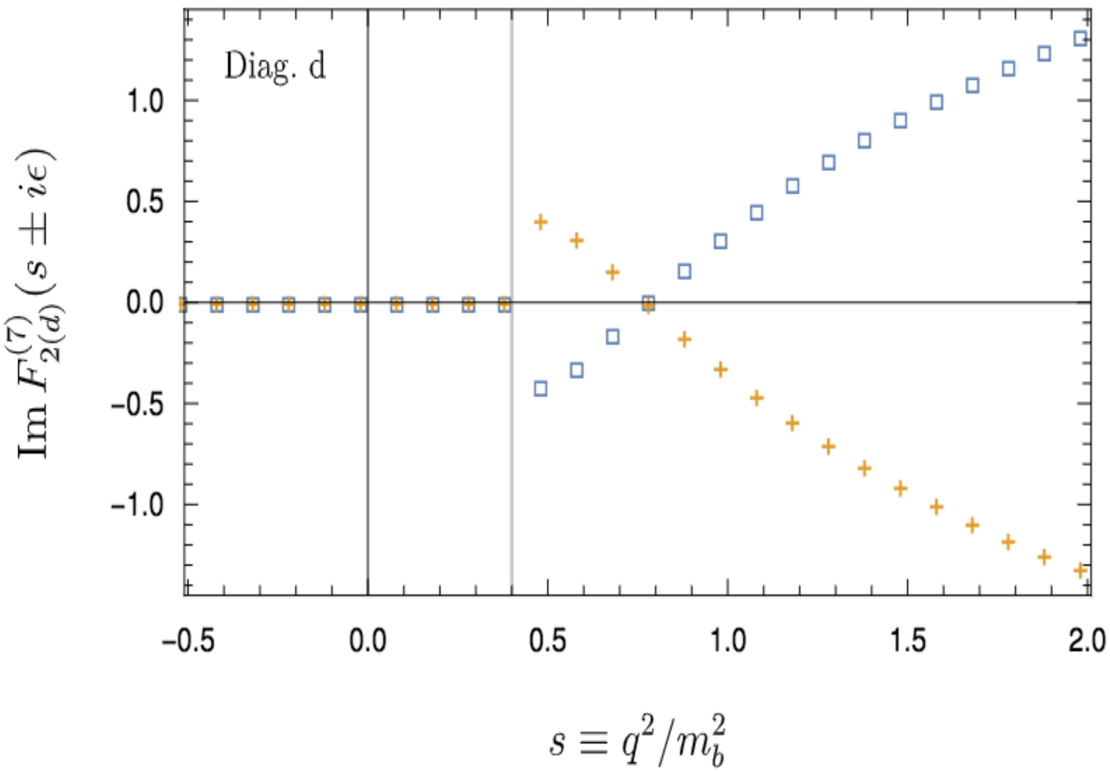
Partonic singularities

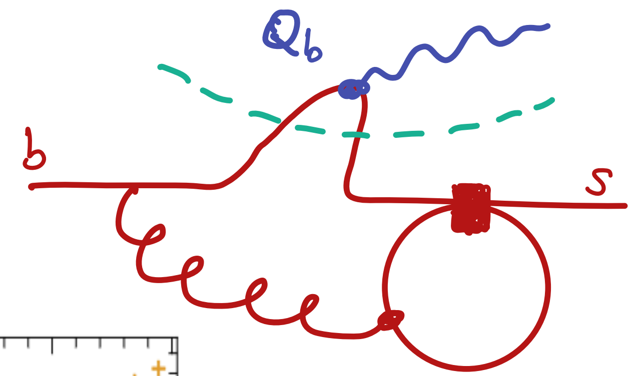
Objective: Fully analytical calculation in two variables: q^2 and m_c.



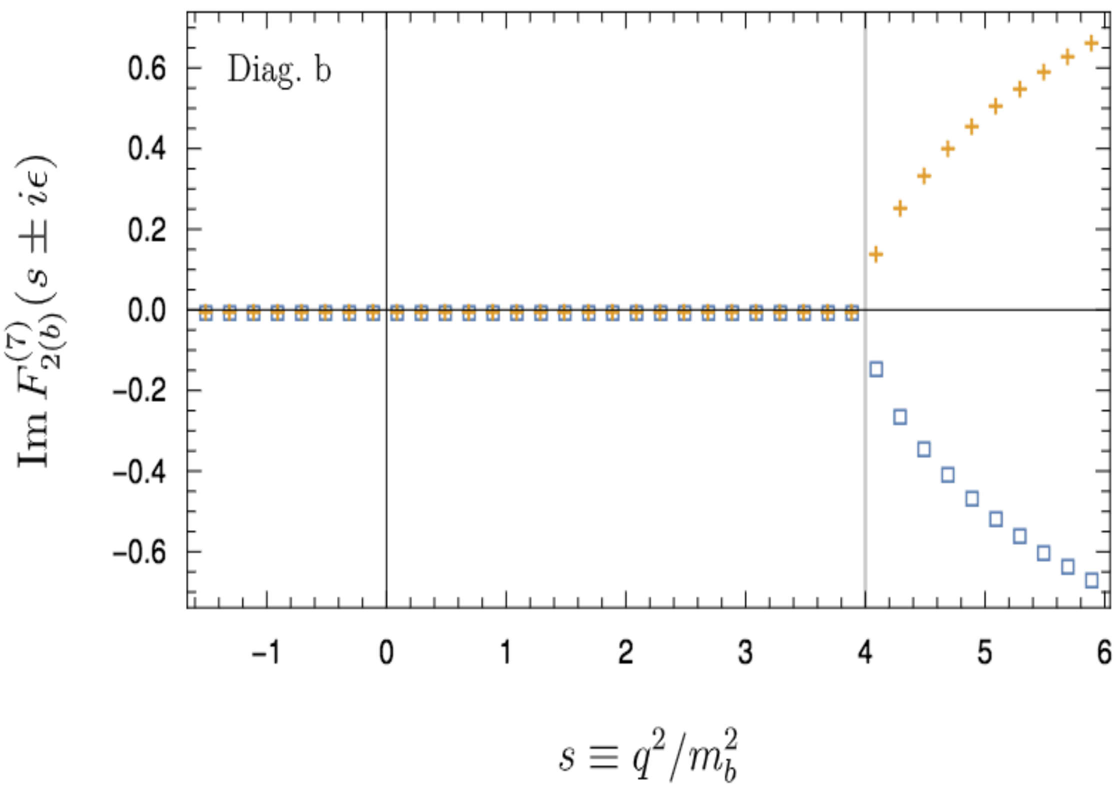


Checking analytic structure of $\mathcal{H}(q^2)$

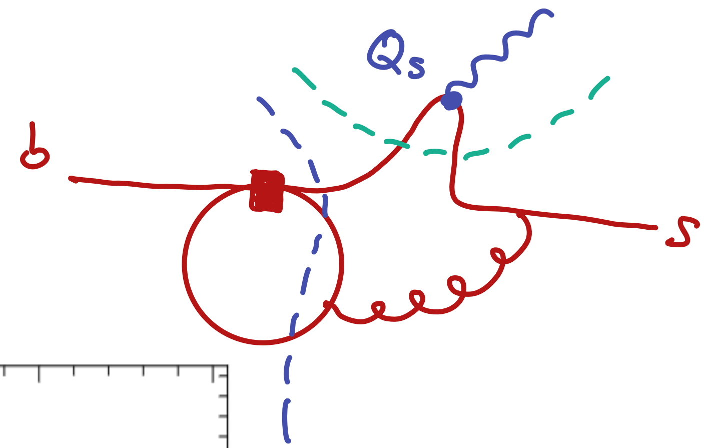




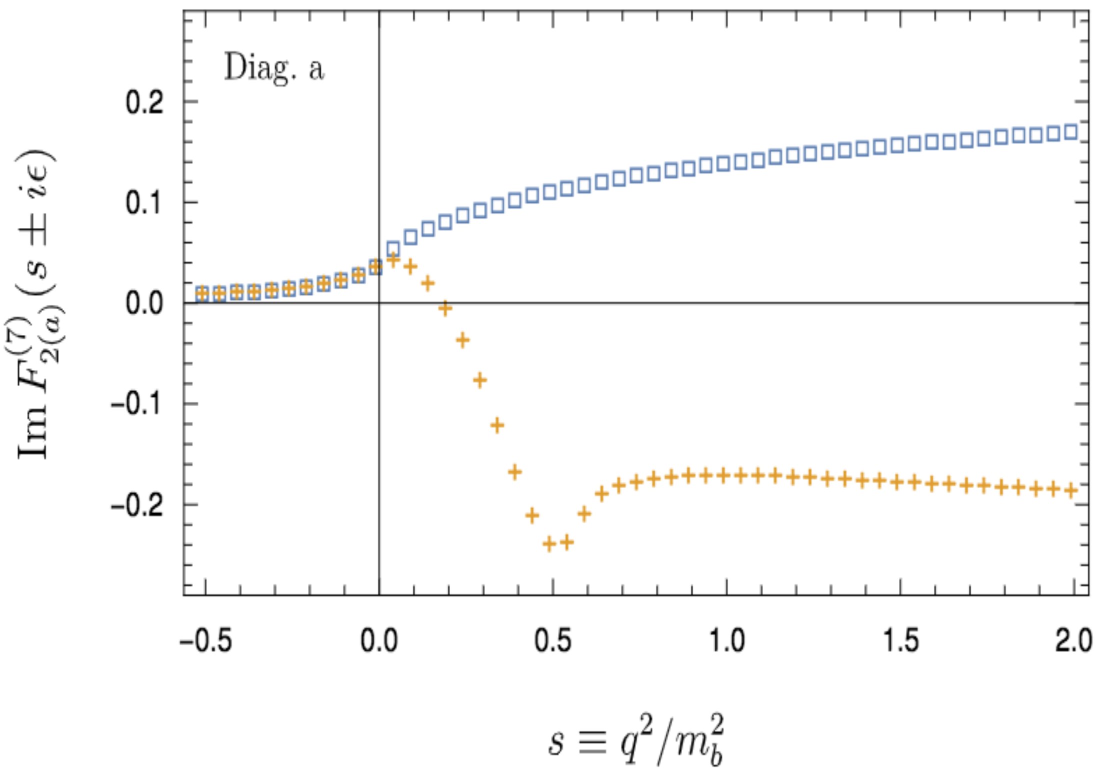
Checking analytic structure of $\mathcal{H}(q^2)$



Asatrian, Greub, Virto 2019



Checking analytic structure of $\mathcal{H}(q^2)$



Dispersive Bound

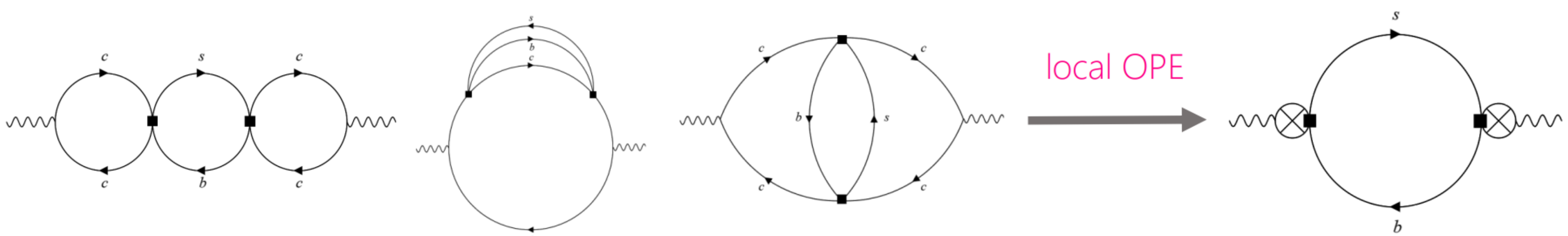
1. Consider the correlation function

$$\Pi(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0| T\left\{O^\mu(q;x), O^{\mu,\dagger}(q;0)\right\} |0\rangle$$

where

$$O^\mu(q;x) = -i \int d^4y\, e^{+iq\cdot y}\; T\{j^\mu_{\rm em}(x+y), (C_1\mathcal{O}_1 + C_2\mathcal{O}_2)(x)\}$$

2. Calculate in OPE region

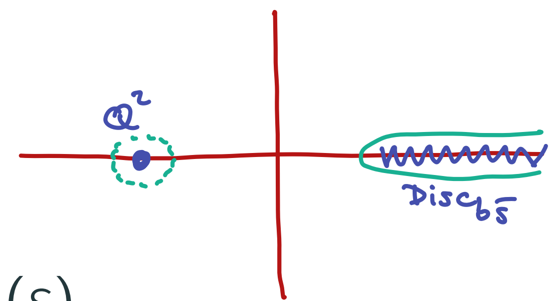


$$\chi^{\rm OPE}(-m_b^2) = (1.81 \pm 0.02) \times 10^4 \mathrm{GeV}^{-2}$$

Dispersive bound

Gubernari, van Dyk, Virto 2020

3. Twice-subtracted dispersion relation:



$$\chi^{\text{OPE}}(Q^2) \equiv \frac{1}{2i\pi} \int_0^\infty ds \, \frac{\text{Disc}_{b\bar{s}} \Pi^{\text{had}}(s)}{(s - Q^2)^3}$$

$$\begin{aligned}
\frac{3}{32 i\pi^3} \text{Disc}_{b\bar{s}} \Pi^{\text{had}}(s) = & \frac{2M_B^4 \, \lambda^{3/2}(M_B^2, M_K^2, s)}{s^4} \left| \mathcal{H}_0^{B\to K}(s) \right|^2 \theta(s - s_{BK}) \\
& + \frac{2M_B^6 \sqrt{\lambda(M_B^2, M_{K^*}^2, s)}}{s^3} \left(\left| \mathcal{H}_\perp^{B\to K^*}(s) \right|^2 + \left| \mathcal{H}_\parallel^{B\to K^*}(s) \right|^2 + \frac{M_B^2}{s} \left| \mathcal{H}_0^{B\to K^*}(s) \right|^2 \right) \theta(s - s_{BK^*}) \\
& + \frac{M_B^6 \sqrt{\lambda(M_{B_s}^2, M_\phi^2, s)}}{s^3} \left(\left| \mathcal{H}_\perp^{B_s\to \phi}(s) \right|^2 + \left| \mathcal{H}_\parallel^{B_s\to \phi}(s) \right|^2 + \frac{M_{B_s}^2}{s} \left| \mathcal{H}_0^{B_s\to \phi}(s) \right|^2 \right) \theta(s - s_{B_s\phi})
\end{aligned}$$

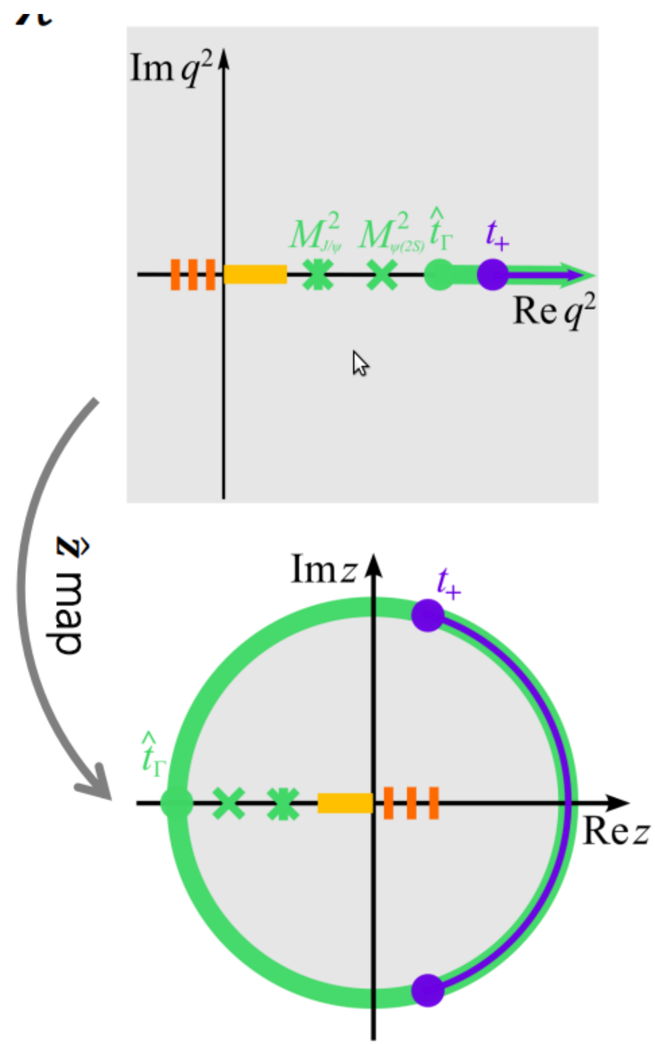
$+$ further positive terms (eg. $\Lambda_b \to \Lambda$, $B \to K\pi\pi\pi\pi\pi\pi$, ...)

Redefine $\mathcal{H}_i$ as before:

$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) \equiv \phi_\lambda^{B\to M}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to M}(z)\,,$$

Expand in ortogonal polynomials in **arc**:

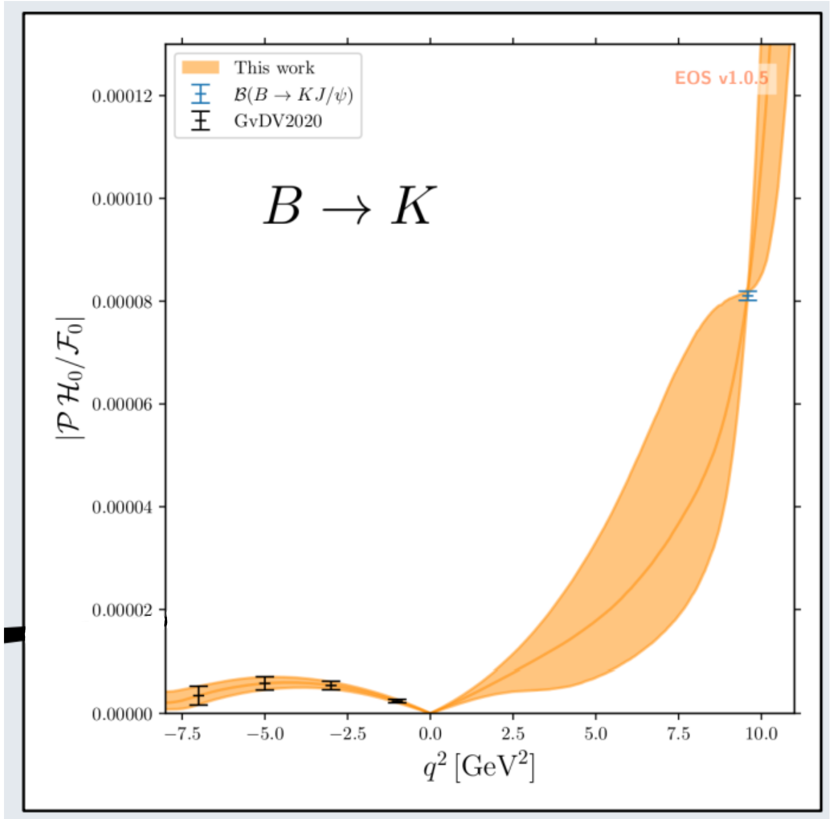
$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M} p_n^{B\to M}(z)$$



The dispersive bound then takes the simple form

$$\sum_{n=0}^{\infty}\left\{2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0}\left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right]\right\} < 1\,.$$

Non-local form factor fitted to LCOPE and J/Ψ data

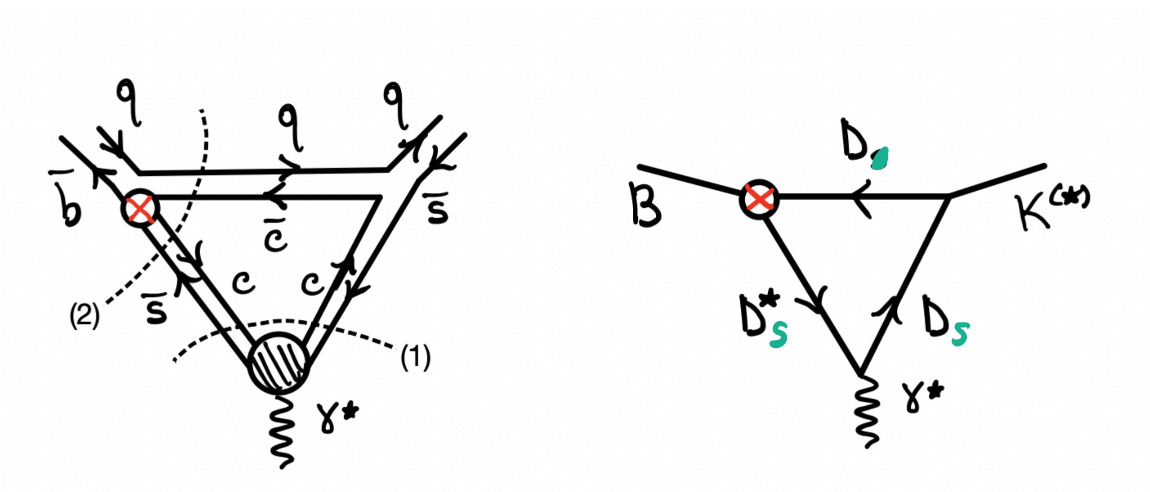


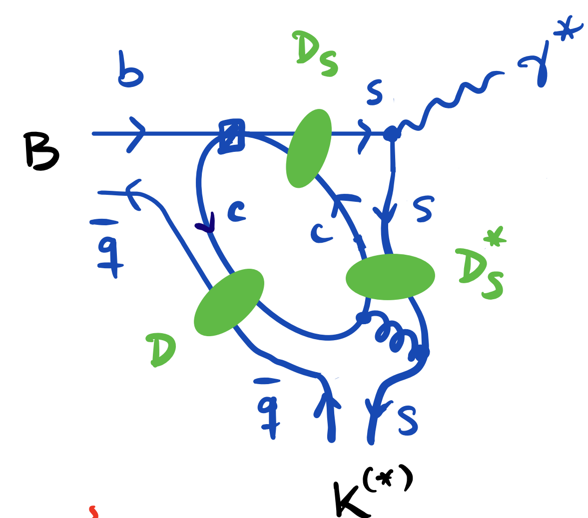
Use under-constrained fit (N = 5) which saturates dispersive bound

All p-values are larger than 11%

Non-Local Form Factors: Issues

- **No LQCD calculation**: need (?) LCSRs
- Leading term is given by Local Form Factors ✓
- $B \to \psi(nS)M \longrightarrow$ important model-independent input (**)
- z-parametrization $\longrightarrow$ **Analytic structure**





(Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli 2022)

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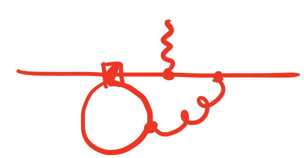
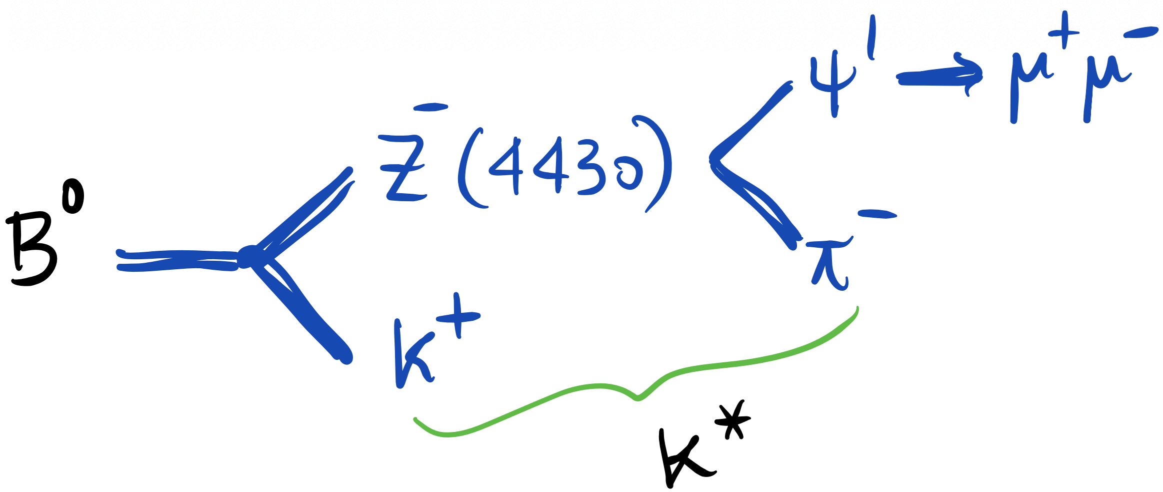
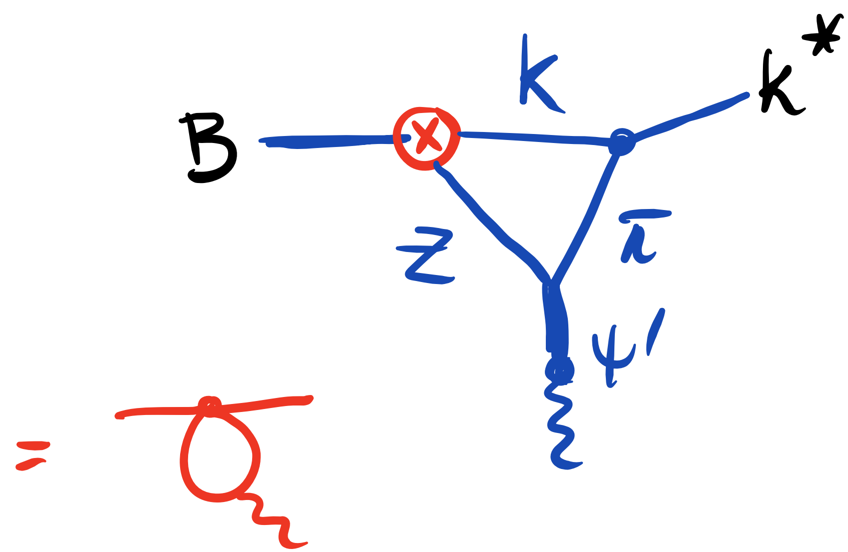


Non-Local Form Factors: Issues

- **No LQCD calculation**: need (?) LCSRs
- Leading term is given by Local Form Factors ✓
- $B \to \psi(nS)M \longrightarrow$ important model-independent input (**)
- z-parametrization $\longrightarrow$ **Analytic structure**





Non-Local Form Factors: Issues

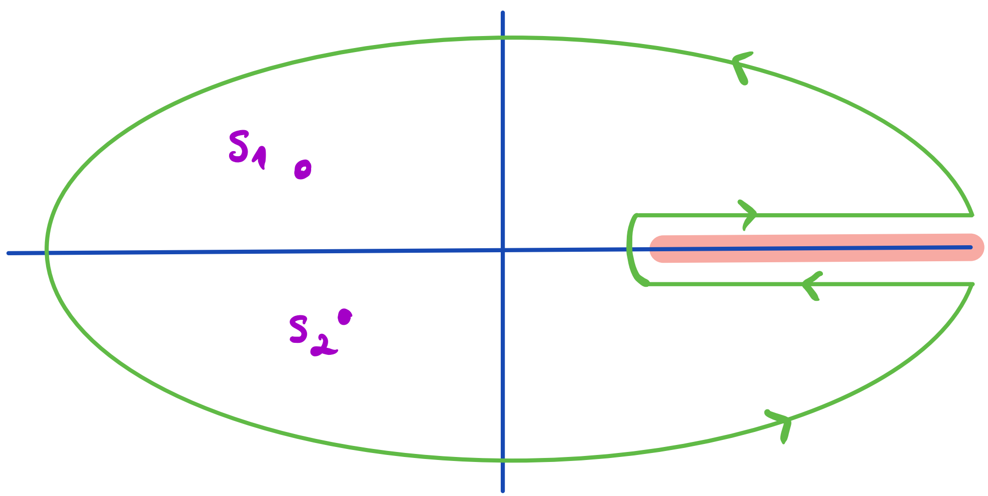
Direct check of analytic structure at two loops: Asatian, Greub, Virto 2019

$$F(s_1) - F(s_2) = \frac{s_1 - s_2}{2\pi i} \int_{s_{\rm th}}^{\infty} dt \frac{F(t+i0) - F(t-i0)}{(t-s_1)(t-s_0)}$$

Example:

$$F^{(7)}_{2,(b)}(-3+i) - F^{(7)}_{2,(b)}(-1-2i) = 0.0894864 - 0.160827\, i\,,$$

$$\frac{-2+3i}{2\pi i}\int_4^\infty dt\, \frac{\text{Disc}\, F^{(7)}_{2,(b)}(t)}{(t+3-i)(t+1+2i)} = 0.0894966 - 0.160839\, i\,.$$

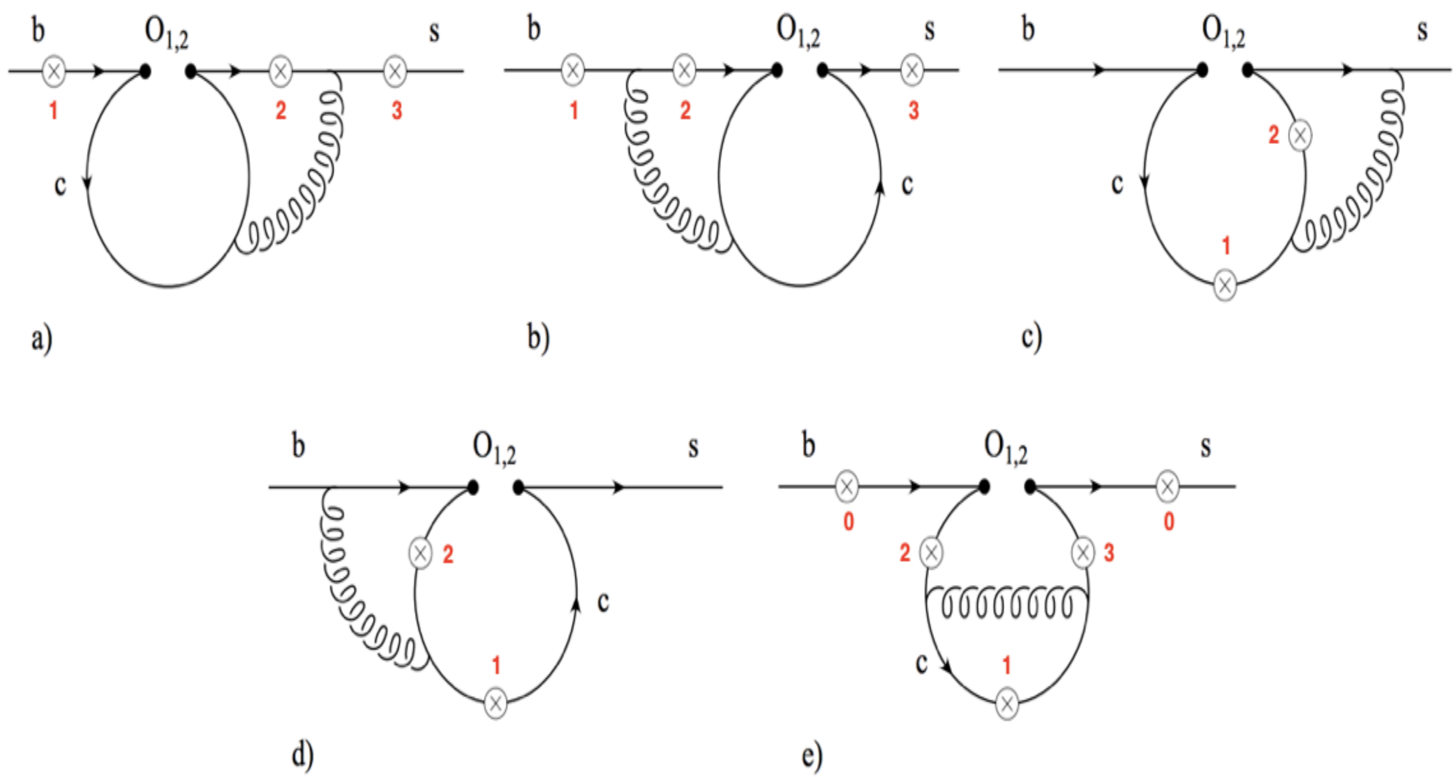


"I summon the spirits of long-distance enhancement"

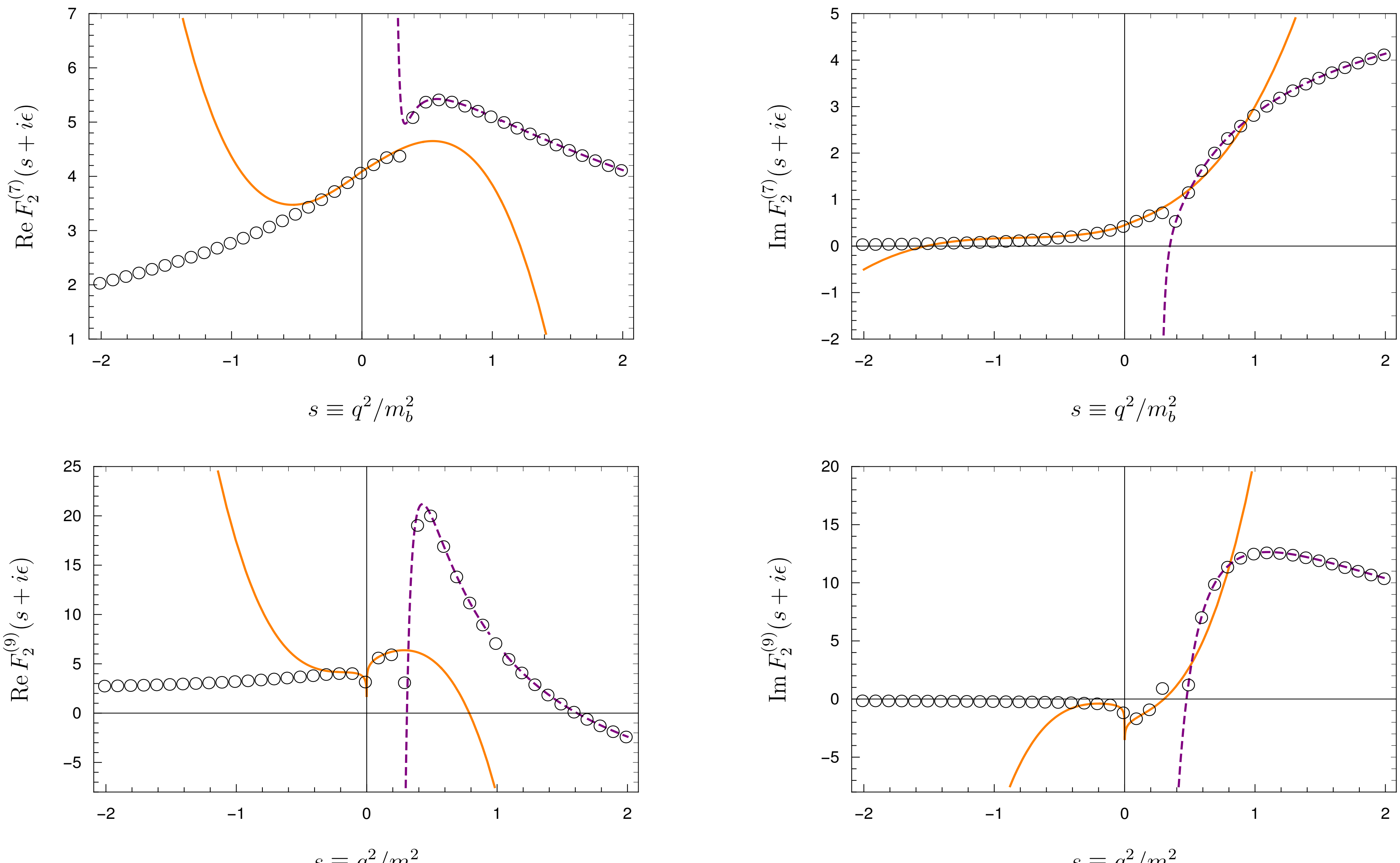
12 Beyond the Flavour Anomalies IV, Barcelona, 21 April 2023 Ulrich Nierste

BACK-UP

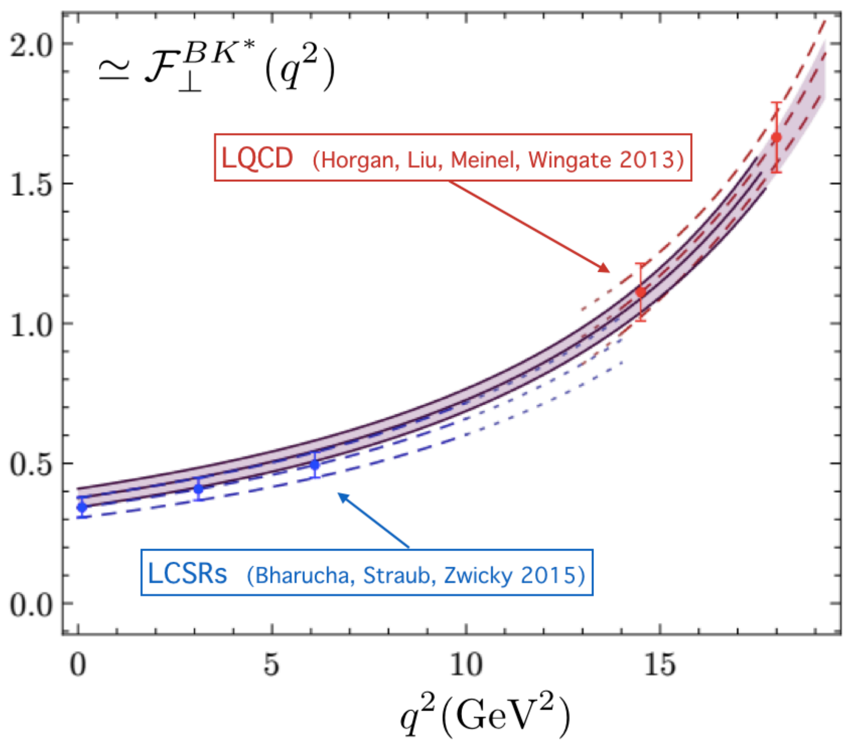
Analytic structure in q^2 plane

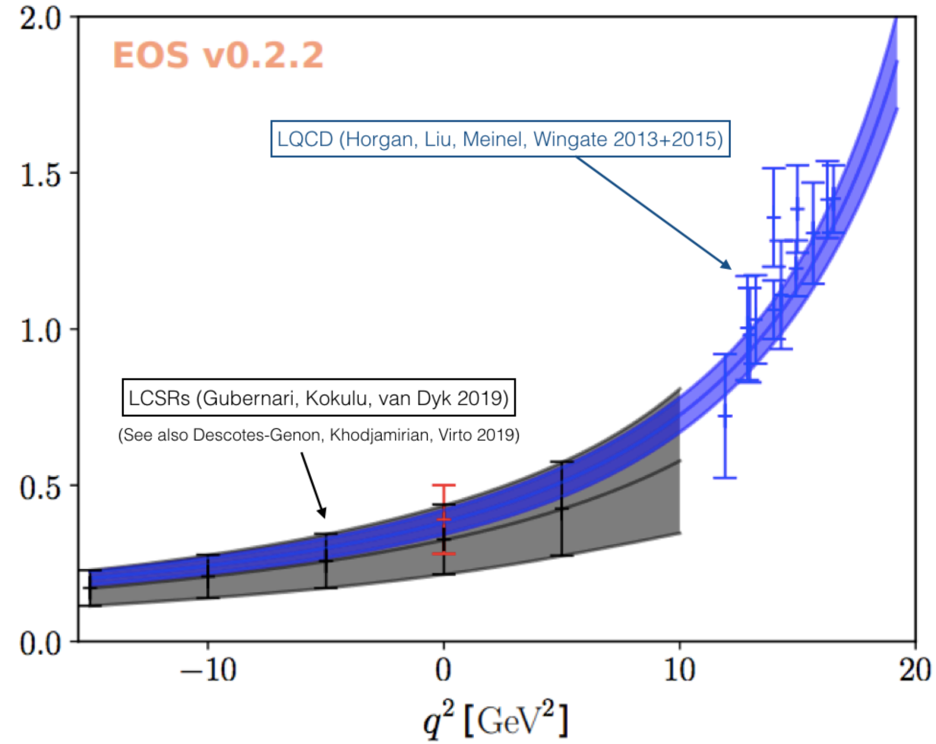


Results: Comparison to previous calculations:



Local Form Factors



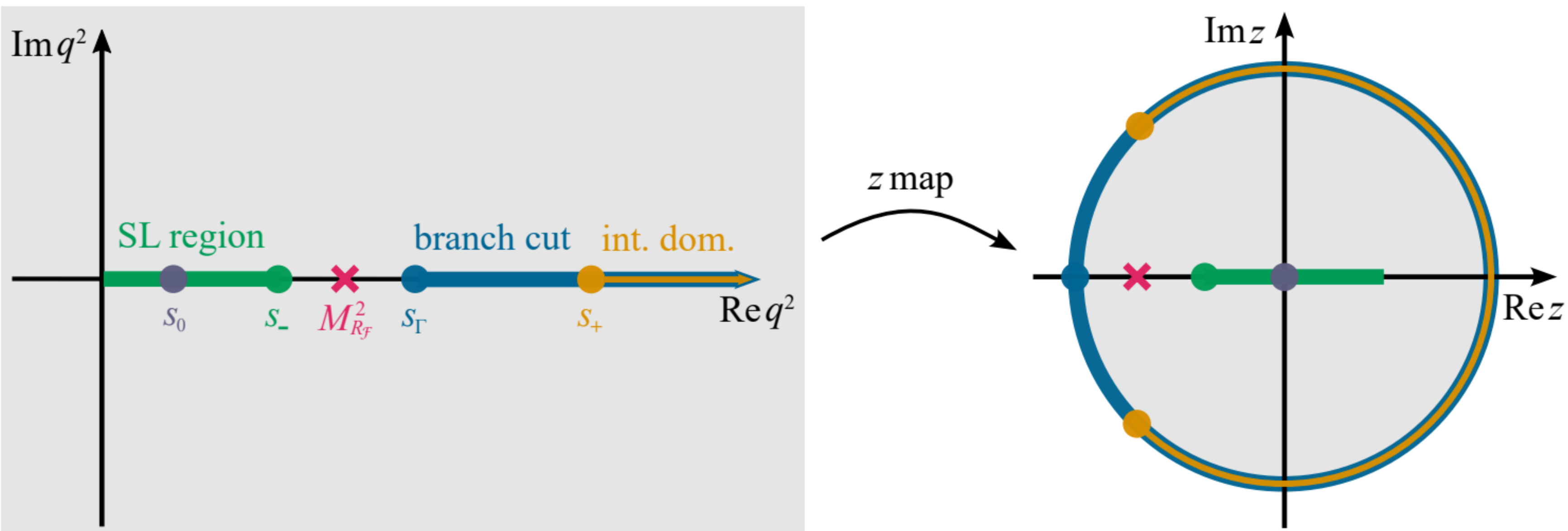


- Two main approaches: (1) Lattice QCD (large q^2 ***) (2) LCSRs (low q^2)
- Two approaches to LCSRs, in terms of (1) K^* LCDAs (2) B LCDAs
- q^2 dependence parametrized via a (dispersively-bounded) z-expansion

Form Factors : q^2-dependence from analyticity

Bourrely, Caprini, Lellouch; Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert; ...

- Conformal mapping : $$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$



- "z-parametrization" : $\widehat{\mathcal{F}}_\lambda^{(T)}(q^2(z))$ is **analytic in** $|z| < 1$ $\quad$ ($|z_{\rm phys}| < 0.15$)

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{(q^2 - m^2_{B_s^*})} \sum_k \alpha_k \, z(q^2)^k$$

Form Factors : Dispersive Bounds (BGL + improvement)

Boyd, Grinstein, Lebed 1997; Bharucha, Feldmann, Wick 2014, Gubernari, Reboud, van Dyk, Virto 2023

1. One starts with the two-point function

$$\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|T\{J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\}|0\rangle = \sum_{\lambda=t,\perp,\|,0} \epsilon_\lambda^\mu \epsilon_\lambda^{\nu *}\, \Pi_\Gamma^{(\lambda)}(q^2)$$

2. The invariant functions fulfil a once-subtracted dispersion relation:

$$\chi_\Gamma^{(\lambda)}(Q^2) = \left[\frac{\partial}{\partial q^2}\right] \Pi_\Gamma^{(\lambda)}(q^2)\Big|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\Pi_\Gamma^{(\lambda)}(s)}{(s-Q^2)^2}\,.$$

3. The function $\chi_\Gamma^{(\lambda)}(Q^2)$ can be calculated in an OPE at a suitable substraction point Q^2
Bharucha, Feldmann, Wick 2014

4. The discontinuity of $\Pi_\Gamma^{(\lambda)}(q^2)$ is the spectral function:

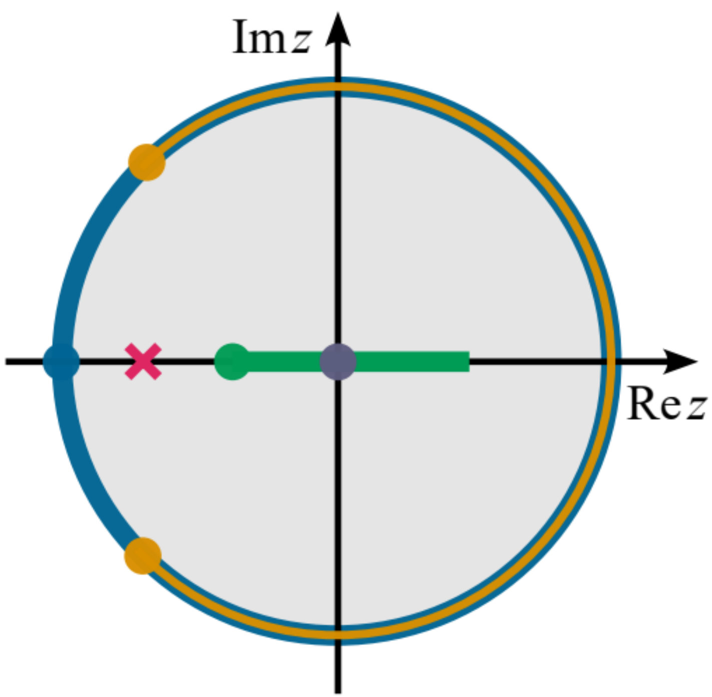
$$\mathrm{Im}\Pi_\Gamma^{(\lambda)}(s) \sim \sum_H \langle 0|J^\mu|H\rangle\langle H|J^{\nu\dagger}|0\rangle \sim f_{B_s^*}^2 + |F^{BK}|^2 + |F^{BK^*}|^2 + |F^{B_s\phi}|^2 + \cdots$$

(up to phase-space functions...)

Flynn, Jüttner, Tsang 2023; Gubernari, Reboud, van Dyk, Virto 2023

In order to simplify the bound, it is thus convenient to reparametrize:

$$\hat{\mathcal{F}}_\lambda^{B\to M}(q^2) = \mathcal{B}_\mathcal{F}(z)\phi_\mathcal{F}(z)\mathcal{F}_\lambda^{B\to M}(q^2) = \sum_k \alpha_k^\mathcal{F}\, p_k^\mathcal{F}(z)$$



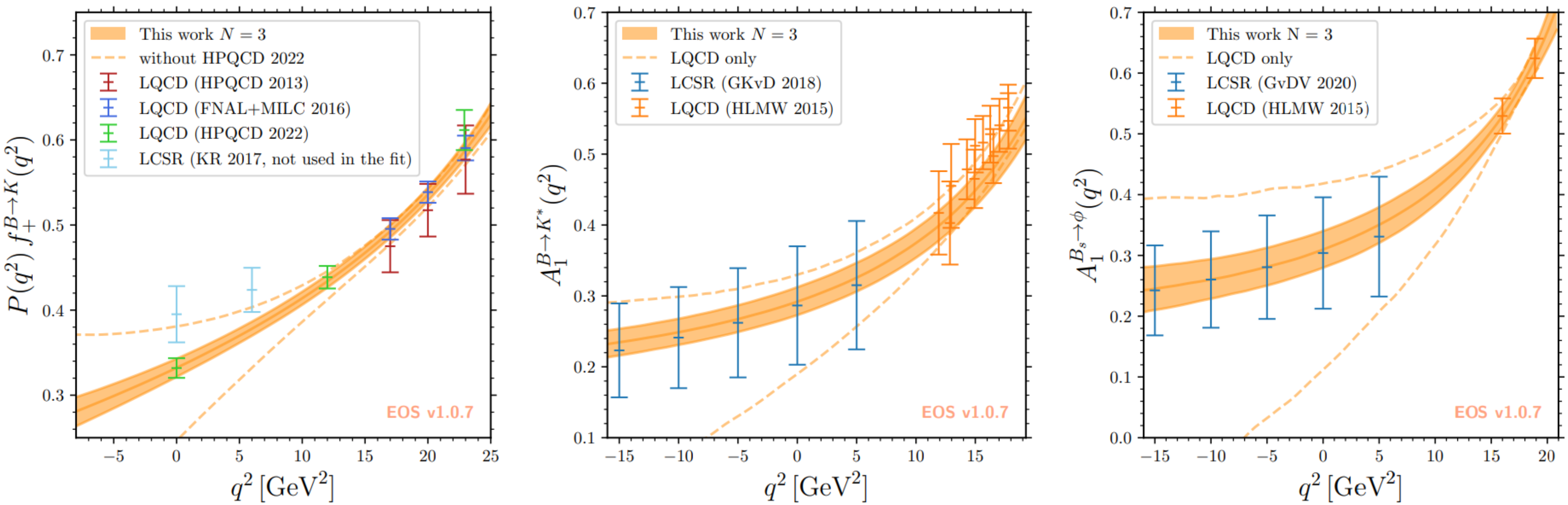
$$\int_{-\alpha_\mathcal{F}}^{+\alpha_\mathcal{F}} d\theta\, p_m^\mathcal{F}(e^{i\theta}) p_n^\mathcal{F}(e^{-i\theta}) = \delta_{mn}$$

$$\sum_{B\to M} \int_{-\alpha_\mathcal{F}}^{+\alpha_\mathcal{F}} d\theta \left|\hat{\mathcal{F}}_\lambda^{B\to M}(e^{i\theta})\right|^2 < 1$$

$$\boxed{\sum_{\mathcal{F},k} |\alpha_k^\mathcal{F}|^2 < 1}$$

Local Form Factors: Results

Gubernari, Reboud, van Dyk, Virto 2023



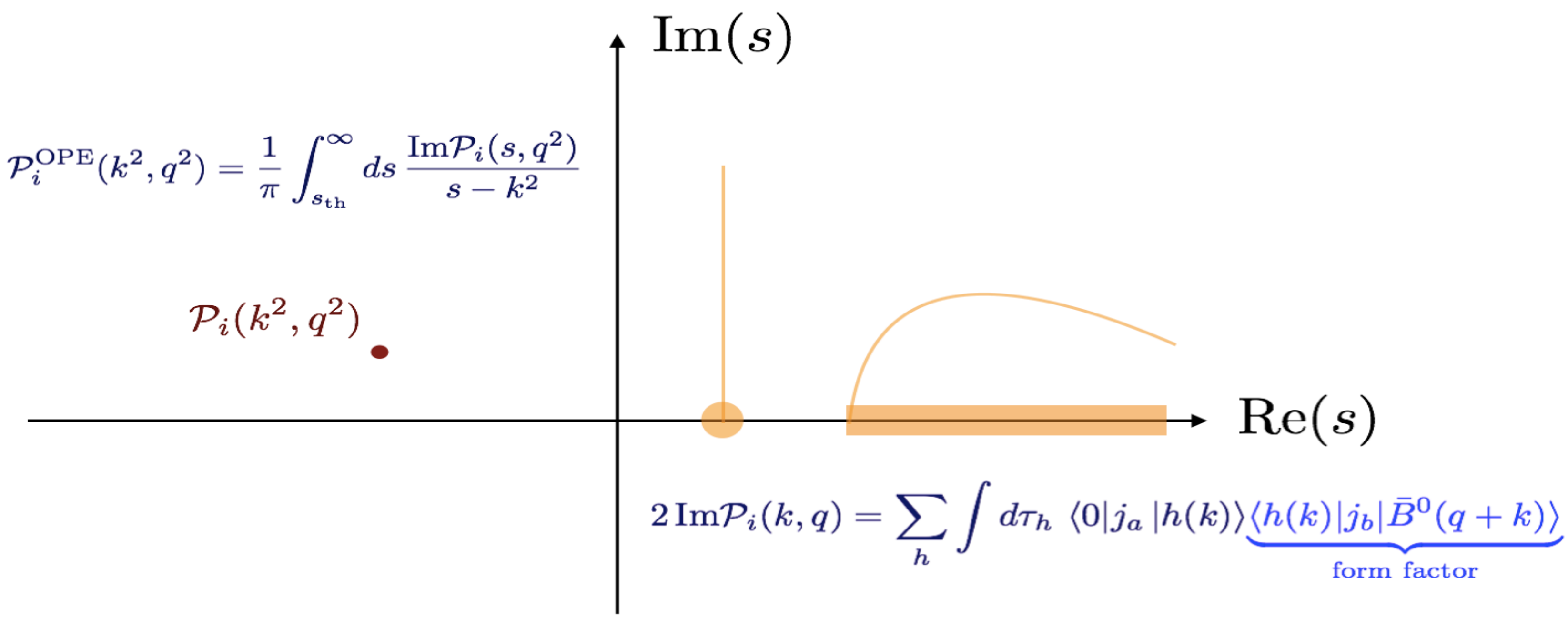
Truncate the series expansion to $N = 2, 3, 4$

Uncertainties stable for $N > 2$

Light-Cone Sum Rules

Khodjamirian, Mannel, Offen 2006 [Unitarity+Analyticity+Duality]

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$



$h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^*\, F^{BK*}\, \delta(k^2 - m_{K^*}) + duality(s_0)$

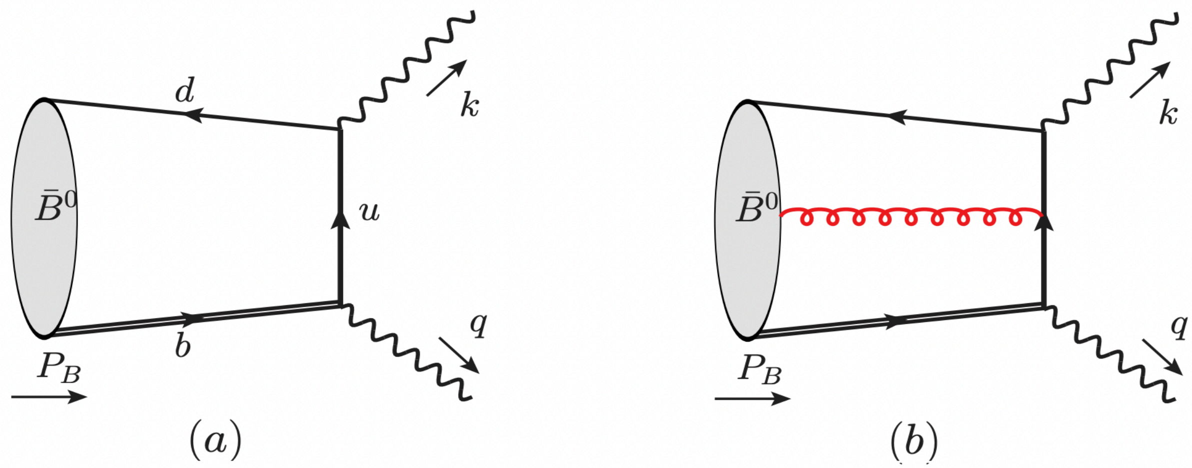
$$F^{BK*}(q^2) = \frac{1}{f_{K^*}\, m_{K^*}} e^{m_{K^*}^2/M^2} \cdot \mathcal{P}^{\mathrm{OPE}}(q^2, s_0, M^2)$$

Kolulu, Gubernari, van Dyk 2018

Descotes-Genon, Khodjamirian, Virto 2019

[Unitarity+Analyticity+Duality]

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$



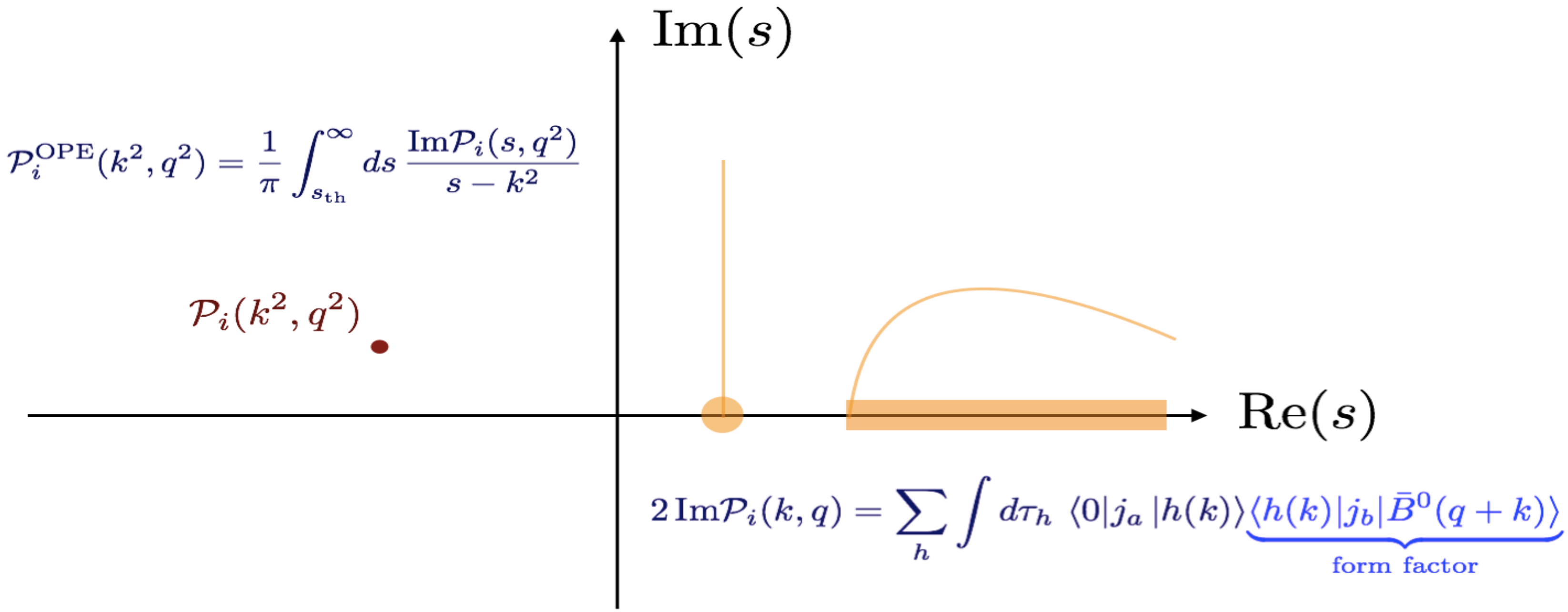
$$\mathcal{P}^{\mathrm{OPE}}(q^2, s_0, M^2) = \sum_{n\geq 0} \frac{f_B m_B}{(M^2)^n} \int_0^{s_0} ds\, e^{-s/M^2} \mathcal{G}_n(s)$$

New calculation includes DAs up to twist-4 Braun, Ji, Manashov 2017

Form Factors: beyond the Narrow Width limit

Cheng, Khodjamirian, Virto 2017; Descotes-Genon, Khodjamirian, Virto 2019

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$



- Traditionally, $h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^* F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$

Form Factors: beyond the Narrow Width limit

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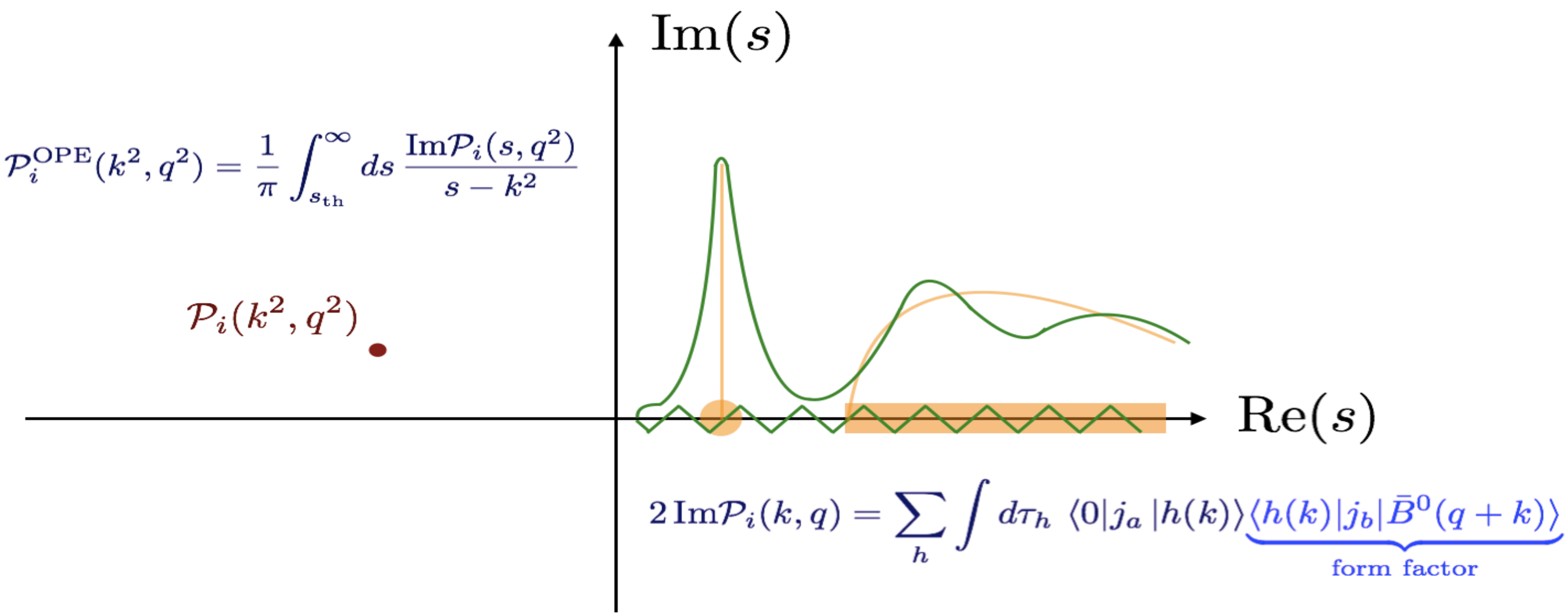


- Traditionally, $h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^* F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$
- Generalization for unstable mesons Cheng, Khodjamirian, Virto 2017 : $h(k) = K\pi + \cdots$

LCSRs with B-meson DAs, natural for this generalization.

LCSRs for P-wave $B \to K\pi$ Form Factors

P-wave Projector

$$\mathcal{P}_i(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{\overbrace{\bar{d}(x)\gamma^\mu s(x)}^{\text{P-wave Projector}}, j_i(0)\}|\bar{B}^0(q+k)\rangle$$

$$\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\;\; \omega_i(s,q^2)\;\; f_+^\star(s)\;\; F_i^{(\ell=1)}(s,q^2) = \mathcal{P}_i^{\rm OPE}(q^2,\sigma_0,M^2)$$

- s_0 – Effective threshold
- $\omega_i(s,q^2)$ – (known) kinematic factors
- $\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma_\mu d|0\rangle = f_+(k^2)\;\bar{k}_\mu + \frac{m_K^2-m_\pi^2}{k^2} f_0(k^2)\; k_\mu$
- $\mathcal{P}_i^{\rm OPE}$ – OPE result for the correlation function

Descotes-Genon, Khodjamirian, Virto 2019

LCSRs for P-wave $B \to K\pi$ Form Factors

$$\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\ \omega_i(s,q^2)\ f_+^\star(s)\ F_i^{(\ell=1)}(s,q^2) = \mathcal{P}_i^{\rm OPE}(q^2,\sigma_0,M^2)$$

- Generalize LCSRs in Khodjamirian, Mannel, Offen 2006 beyond the K^*, including LCSRs for A_0, $T_{2,3}$
- Recalculate $\mathcal{P}_i^{\rm OPE}$ including 3-particle contributions, and extended consistently to twist-4 accuracy. Full (numerical) agreement with Gubernari,Kokulu,van Dyk 2018 (not input parameters)
- Revisit s_0 $\Rightarrow$ significantly lower value!! – f_{K^*} is derived quantity
- Study of Narrow-width limit, Finite-Width effects, and effects beyond the K^*
- Applications to $B \to K\pi\ell\ell$

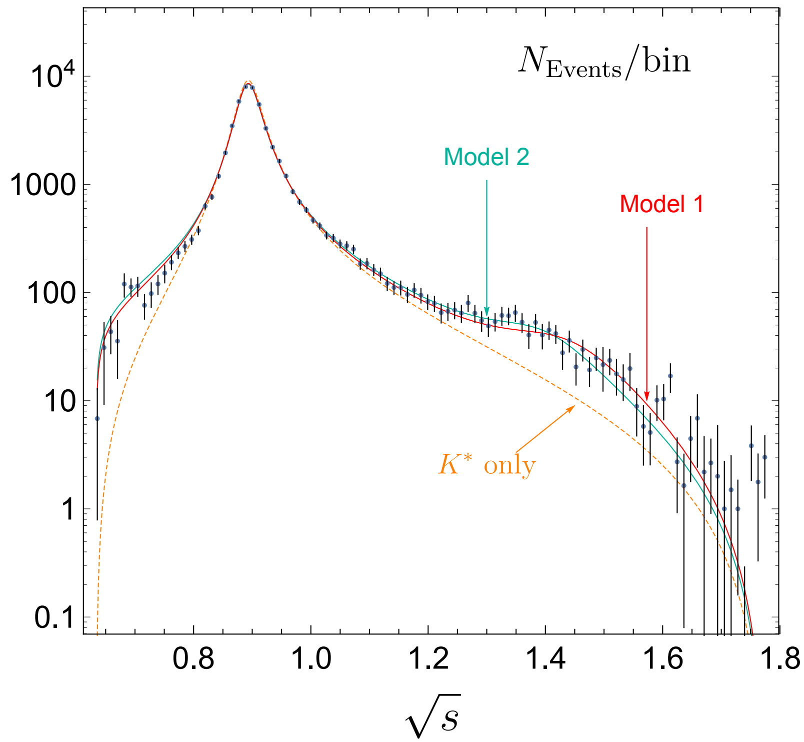
Descotes-Genon, Khodjamirian, Virto 2019

$$\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\; \omega_i(s,q^2)\; f_+^\star(s)\; F_i^{(\ell=1)}(s,q^2) = \mathcal{P}_i^{\rm OPE}(q^2,\sigma_0,M^2)$$

$$f_+^\star(s)\; F_i^{(\ell=1)}(s,q^2) \longrightarrow f_K^*\, \mathcal{F}_{R,i}(q^2)\, \delta(s-m_K^*)$$

$$f_+(s) = -\sum_R \frac{m_R\, f_R\, g_{RK\pi}\; e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

$$F_i^{(\ell=1)}(s,q^2) = \sum_R \frac{Y_{R,i}(s,q^2)\, g_{RK\pi}\, \mathcal{F}_{R,i}(q^2)\, e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$



Descotes-Genon, Khodjamirian, Virto 2019

LCSRs for P-wave $B \to K\pi$ Form Factors: Narrow-Width Limit

Khodjamirian Mannel Offen 2006 — KMPW 2010 — Gubernari Kokulu van Dyk 2018 — BSZ 2015

Form Factor	This work	Ref. [12]	Ref. [24]	Ref. [15]	Ref. [17]
$\mathcal{F}_{K^*,\perp}(0) = V^{BK^*}(0)$	0.26(15)	0.39(11)	0.36(18)	0.32(11)	0.34(4)
$\mathcal{F}_{K^*,\parallel}(0) = A_1^{BK^*}(0)$	0.20(12)	0.30(8)	0.25(13)	0.26(8)	0.27(3)
$\mathcal{F}_{K^*,-}(0) = A_2^{BK^*}(0)$	0.14(13)	0.26(8)	0.23(15)	0.24(9)	0.23(5)
$\mathcal{F}_{K^*,t}(0) = A_0^{BK^*}(0)$	0.30(7)	–	0.29(8)	0.31(7)	0.36(5)
$\mathcal{F}^T_{K^*,\perp}(0) = T_1^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,\parallel}(0) = T_2^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,-}(0) = T_3^{BK^*}(0)$	0.13(12)	–	0.22(14)	0.20(8)	0.18(3)

Table 6: *Results for the form factors at $q^2 = 0$ in the narrow-width limit,compared to corresponding results in the literature. The approach in Ref. [17] is a completely different LCSR approach, in terms of K^* DAs.*

Descotes-Genon, Khodjamirian, Virto 2019

LCSRs for P-wave $B \to K\pi$ Form Factors: Narrow-Width Limit

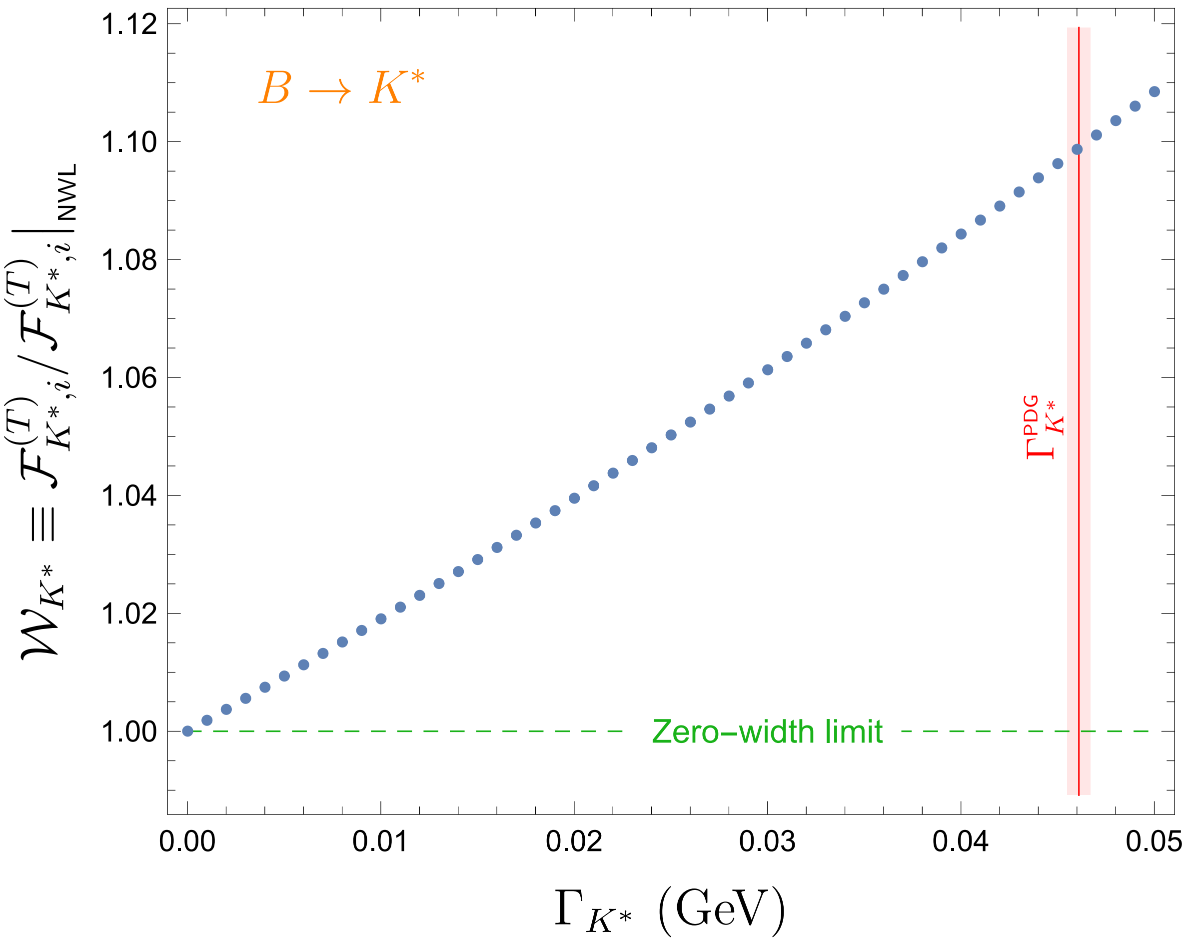
$\mathcal{F}^{BK^*}(q^2=0)$	V^{BK^*}	$A_1^{BK^*}$	$A_2^{BK^*}$	$A_0^{BK^*}$	$T_{1,2}^{BK^*}$	$T_3^{BK^*}$
Ref. [12]	0.39	0.30	0.26	–	0.33	–
Inputs [12], no g_+	0.38	0.29	0.26	0.31	0.33	0.25
Inputs [12], with g_+	0.27	0.21	0.14	0.31	0.24	0.14
Our inputs, but $s_0 = 1.7\,\text{GeV}^2$	0.33	0.26	0.17	0.38	0.29	0.17
Our inputs, our s_0, no g_+	0.36	0.28	0.25	0.30	0.31	0.23
Our inputs, our s_0, with g_+	0.26	0.20	0.14	0.30	0.22	0.13

Table 7: *Deconstruction of the different effects explaining the difference between our results for the form factors at $q^2 = 0$ and those in Ref. [12]. The difference stems mainly from the inclusion of the twist-four two-particle contributions. See the text for more details.*

2pt Twist-4 LCDA

Descotes-Genon, Khodjamirian, Virto 2019

Finite-width effects



$$\mathcal{W}_{K^*} \simeq 1 + 1.9\,\frac{\Gamma_{K^*}}{m_{K^*}}$$

$$\mathcal{W}_{K^*} = 1.09 \pm 0.01$$

$\Rightarrow$ BRs are corrected by a factor $|\mathcal{W}_{K^*}|^2 \simeq 1.2$

Descotes-Genon, Khodjamirian, Virto 2019

Consider the sum rule with $R = \{K^*(892), K^*(1410)\}$:

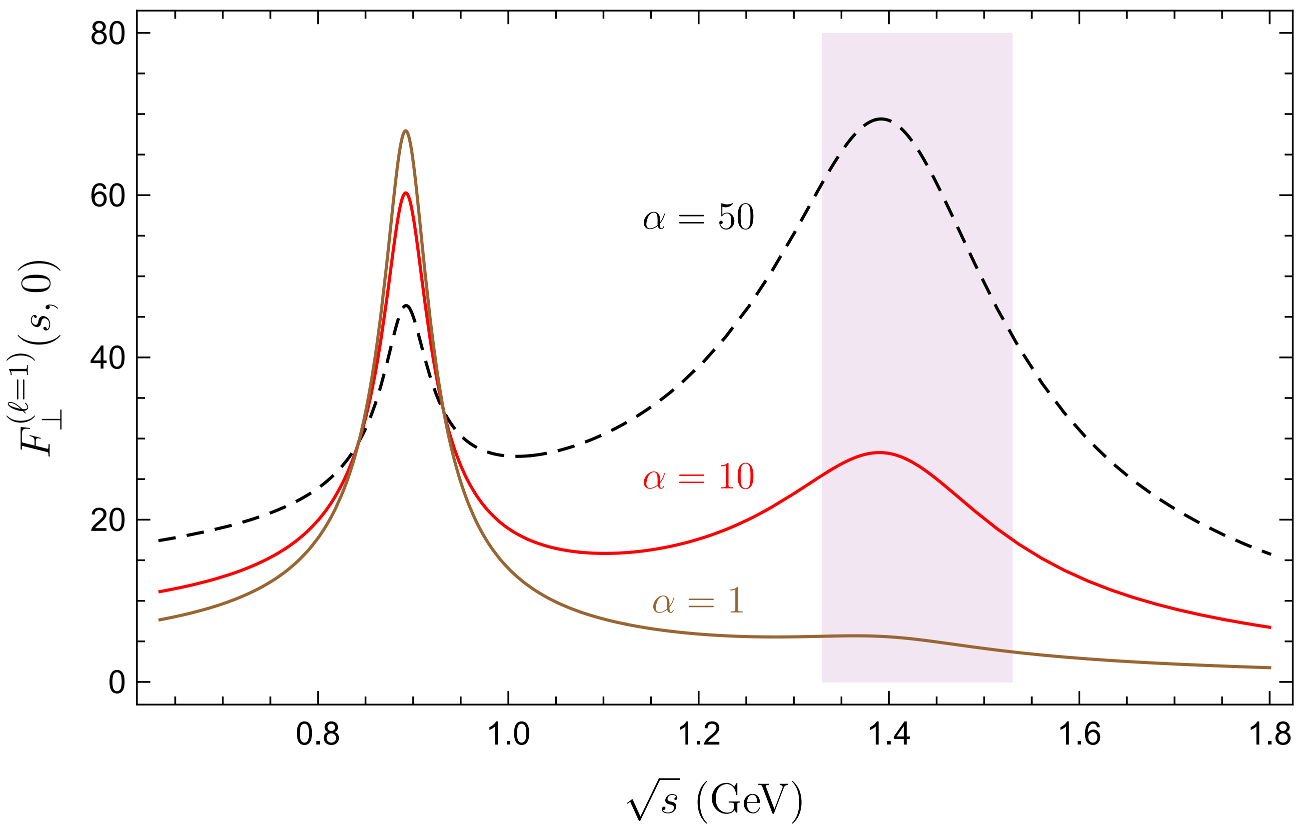
$$\sum_R \mathcal{F}_{R,i}^{(T)}(q^2)\, d_{R,i}^{(T)}\, I_R(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)$$

		$M^2 = 1.00\,\mathrm{GeV}^2$	$M^2 = 1.25\,\mathrm{GeV}^2$	$M^2 = 1.50\,\mathrm{GeV}^2$
Model 1	$I_{K^*(892)}$	0.1506(23)	0.1781(16)	0.1992(13)
	$I_{K^*(1410)}$	0.0050(07)	0.0062(07)	0.0072(06)
Model 2	$I_{K^*(892)}$	0.1491(22)	0.1766(20)	0.1975(16)
	$I_{K^*(1410)}$	0.0048(07)	0.0061(06)	0.0070(06)

Table 8: *Values for the quantities I_R for $R = \{K^*(892), K^*(1410)\}$ for the different values of the Borel parameter M^2 and for the two models for the $K\pi$ form factor. The $K^*(1410)$ contribution is very suppressed in the sum rules, with $I_{K^*(1410)}/I_{K^*(892)} \simeq 0.03$ in all cases.*

Descotes-Genon, Khodjamirian, Virto 2019

Set $\mathcal{F}_{K^*(1410)} = \alpha\,\mathcal{F}_{K^*(892)}$ with α a floating parameter



$\alpha = 1: \ \mathcal{F}_{K^*,\perp}(0) = 0.28\,; \quad \alpha = 10: \ \mathcal{F}_{K^*,\perp}(0) = 0.22\,; \quad \alpha = 50: \ \mathcal{F}_{K^*,\perp}(0) = 0.11\,.$

Descotes-Genon, Khodjamirian, Virto 2019

Beyond the $K^*(892)$

Descotes-Genon, Khodjamirian, Virto, Vos 2023

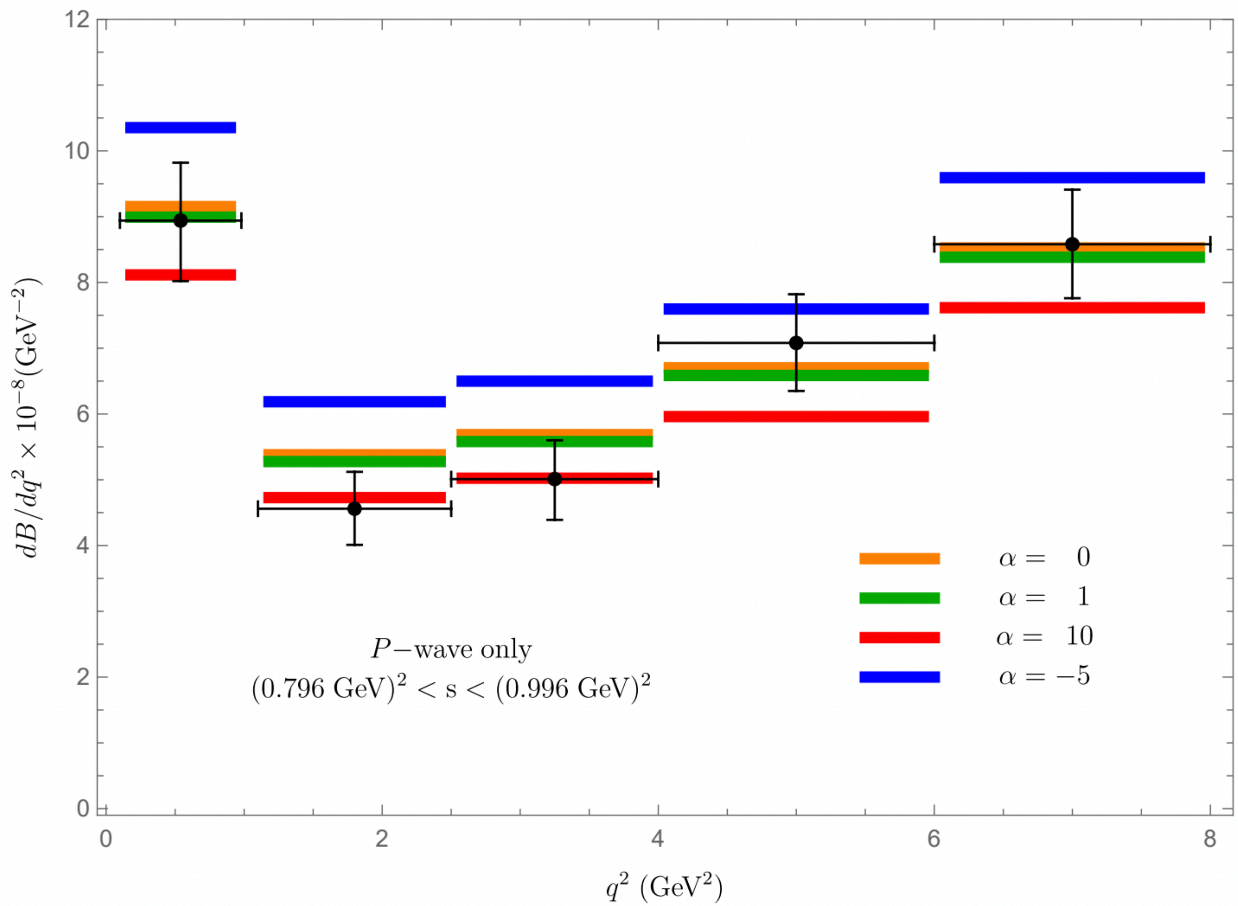


Figure 8: *Theory predictions for the* $B \to (K\pi)_P \ell^+\ell^-$ *branching ratio within the* $K\pi$ *invariant mass bin* $(0.796\ \text{GeV})^2 < s < (0.996\ \text{GeV})^2$, *for different values of* α, *compared to the LHCb measurements of* $B \to K^*\mu^+\mu^-$ *in Ref. [13].*

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

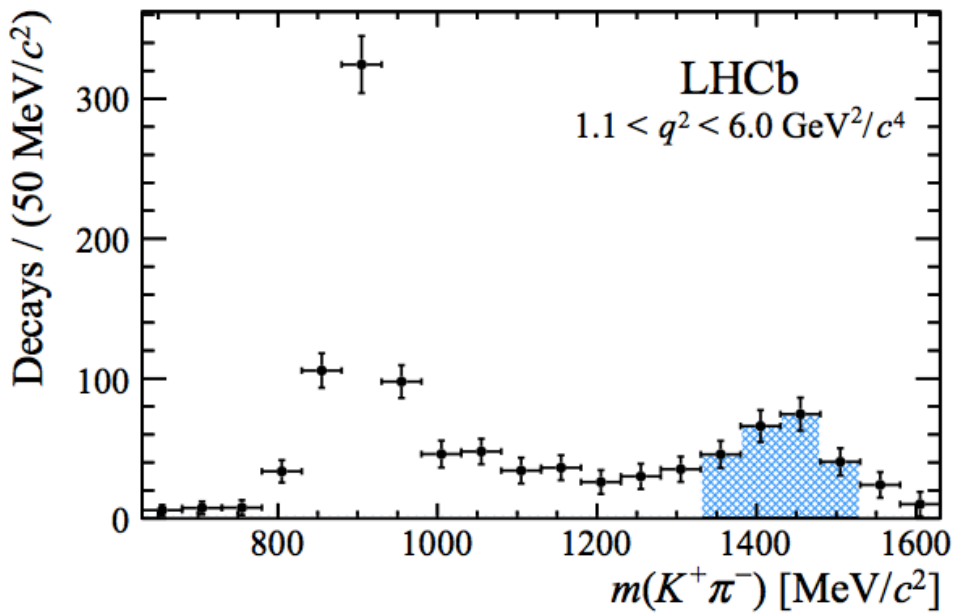
LHCb arXiv: 1609.04736

Differential decay rate including S,P,D waves – – [$d\Omega = d\cos\theta_\ell \, d\cos\theta_K \, d\phi$]

$$\frac{d\Gamma}{dq^2 dk^2 d\Omega} = \frac{1}{4\pi} \sum_{i=1}^{41} f_i(\Omega) \, \tilde{\Gamma}_i(q^2, k^2)$$

The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ have been measured by LHCb (arXiv: 1609.04736) in the bins

$$\sqrt{k^2} \in [1.33, 1.53]\text{GeV} \; , \quad q^2 \in [1.1, 6]\text{GeV}^2$$



High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

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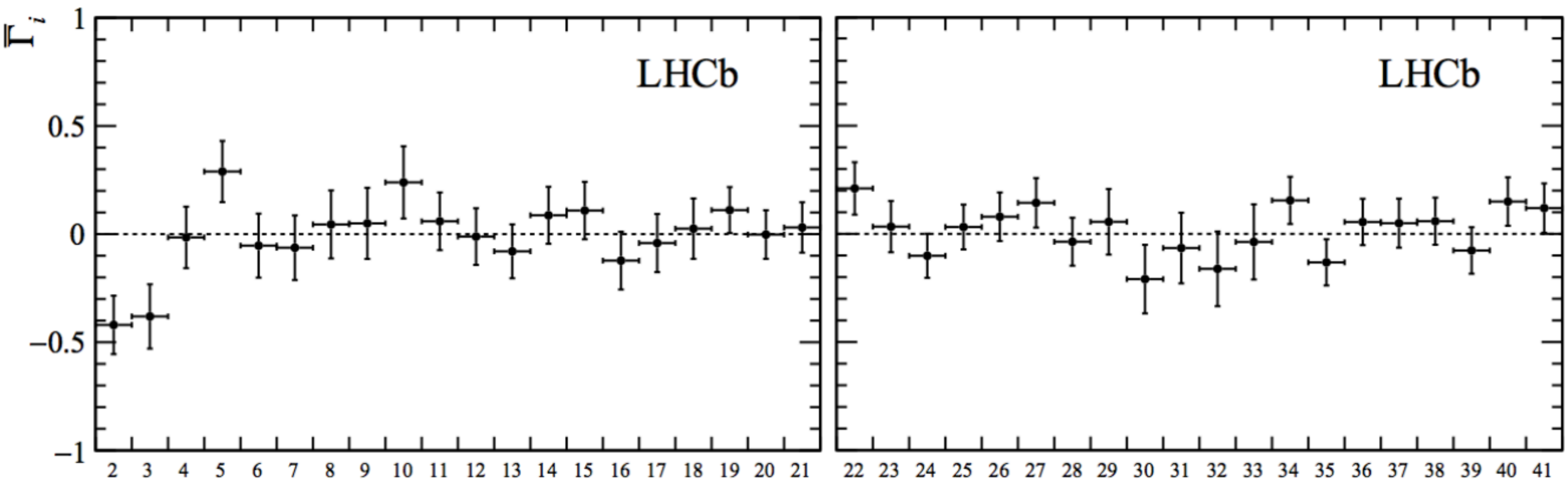
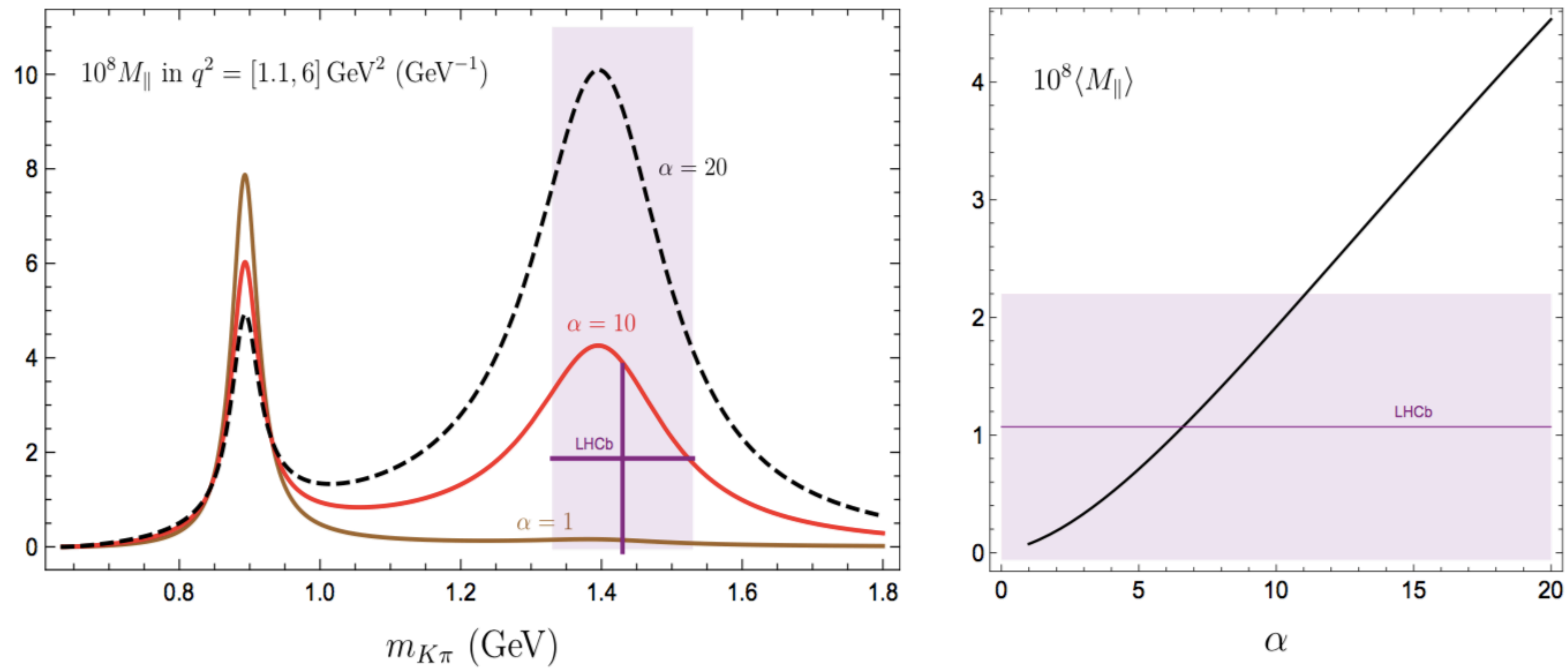


High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Example: $\langle M_{\|} \rangle \equiv \tau_B \langle |\widehat{A}^L_{\|}|^2 + |\widehat{A}^R_{\|}|^2 \rangle = \frac{\tau_B}{36} \langle 5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 - 5\sqrt{15}\tilde{\Gamma}_{19} + 35\sqrt{3}\tilde{\Gamma}_{21} \rangle$



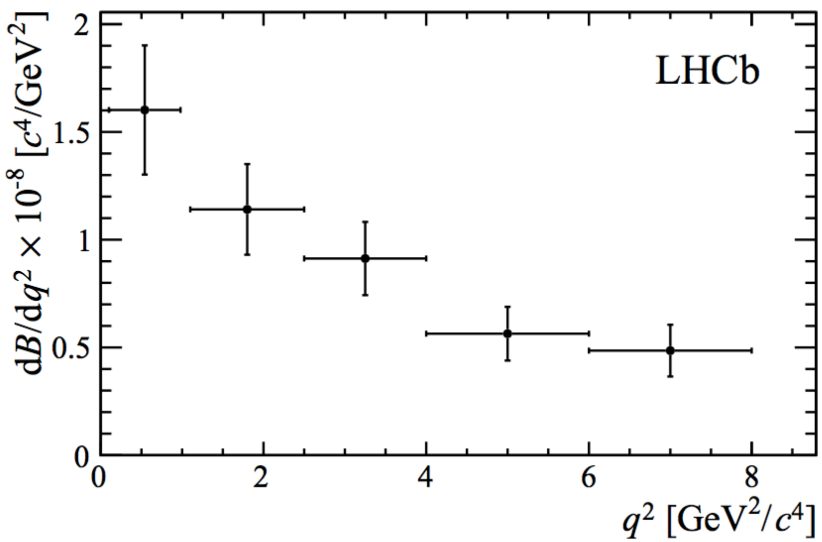
Bounds: From $\langle M_{\|} \rangle$: $\alpha \lesssim 11$; From $\langle M_{\perp} \rangle$: $\alpha \lesssim 17$; From $\langle M_{\rm re} \rangle$: $\alpha \lesssim 18$.

Descotes-Genon, Khodjamirian, Virto 2019

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Upper bounds on P-wave from differential BR:

$$\frac{d\Gamma}{dq^2 dk^2} = \tilde{\Gamma}_1 = |\widehat{A}^L_{\|}|^2 + |\widehat{A}^R_{\|}|^2 + |\widehat{A}^L_{\perp}|^2 + |\widehat{A}^R_{\perp}|^2 + |\widehat{A}^L_0|^2 + |\widehat{A}^R_0|^2 + \dots$$



$$\begin{aligned}
10^8 \cdot \langle \mathcal{B} \rangle_{[0.10,0.98]} &= 1.41 \pm 0.27 \to \ \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[1.10,2.50]} &= 1.60 \pm 0.29 \to \ \alpha \lesssim 6 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[2.50,4.00]} &= 1.37 \pm 0.26 \to \ \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[4.00,6.00]} &= 1.12 \pm 0.26 \to \ \alpha \lesssim 4 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[6.00,8.00]} &= 0.98 \pm 0.23 \to \ \alpha \lesssim 3
\end{aligned}$$

Bounds are easily improved with some info on S-wave form factors.

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S-wave Projector

$$\mathcal{S}_i(k,q) = i \int d^4x\, e^{ik\cdot x} \langle 0 | \mathrm{T}\{\overbrace{\bar{d}(x)s(x)}, j_i(0)\} | \bar{B}^0(q+k)\rangle$$

$$\boxed{\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\ \omega_i(s,q^2)\ f_0^\star(s)\ F_i^{(\ell=0)}(s,q^2) = \mathcal{S}_i^{\rm OPE}(q^2,\sigma_0,M^2)}$$

- s_0 – Effective threshold
- $\omega_i(s,q^2)$ – (known) kinematic factors
- $\langle K^-(k_1)\pi^+(k_2)|\bar{s}d|0\rangle = (m_K^2 - m_\pi^2) f_0(k^2)$
- $\mathcal{S}_i^{\rm OPE}$ – OPE result for the correlation function

LCSRs for S-wave $B \to K\pi$ Form Factors

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Modelling S-wave spectrum much more challenging

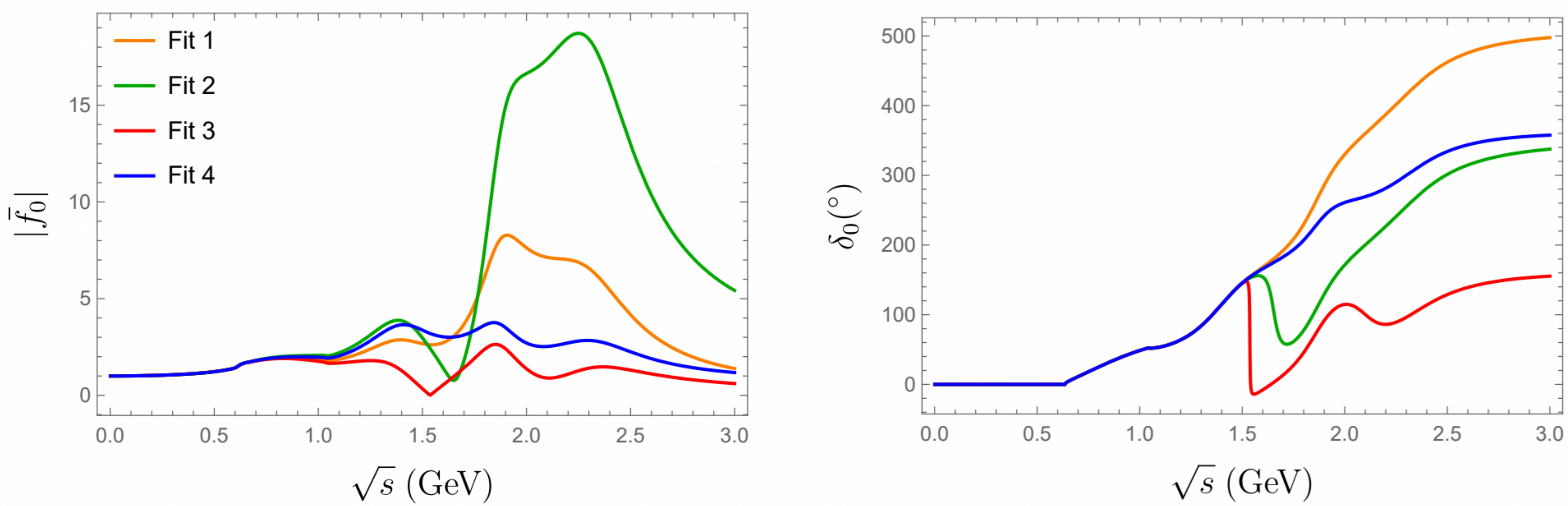

Figure 1: *Modulus of the normalized scalar form factor $|\bar{f}_0|$ and its strong phase δ_0 obtained from the four different fit scenarios of Ref. [30].*

Two channel $K\pi - K\eta'$ model from Von Detten, Noel, Hanhart, Hoferichter, Kubis 2021

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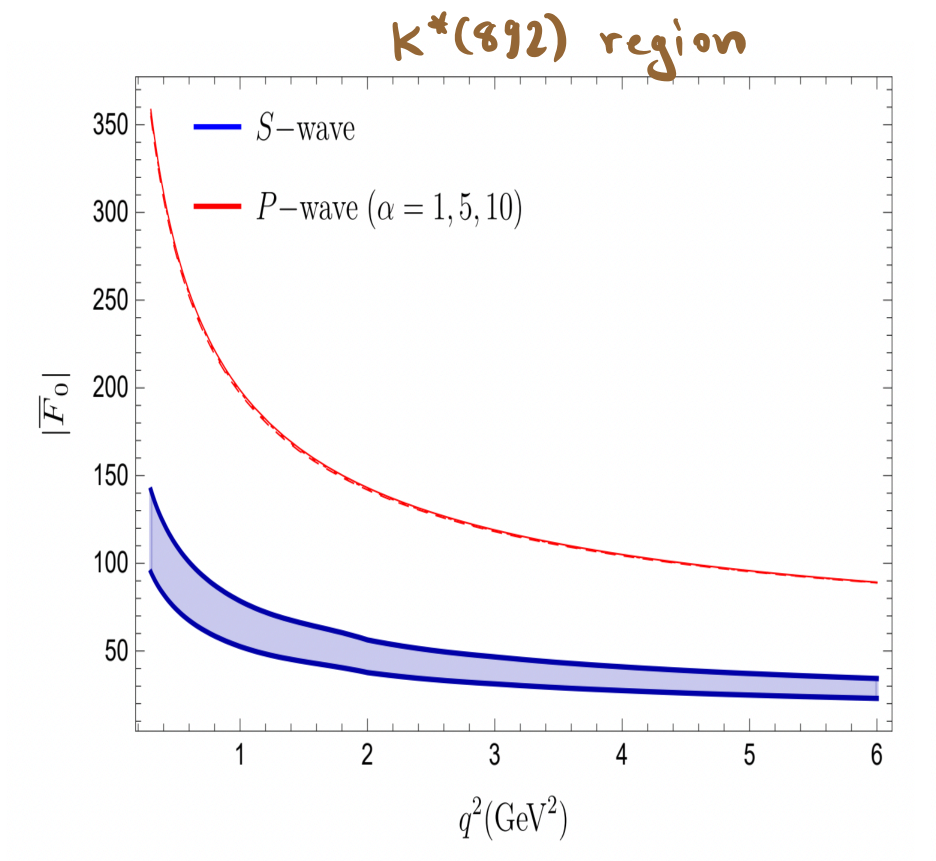
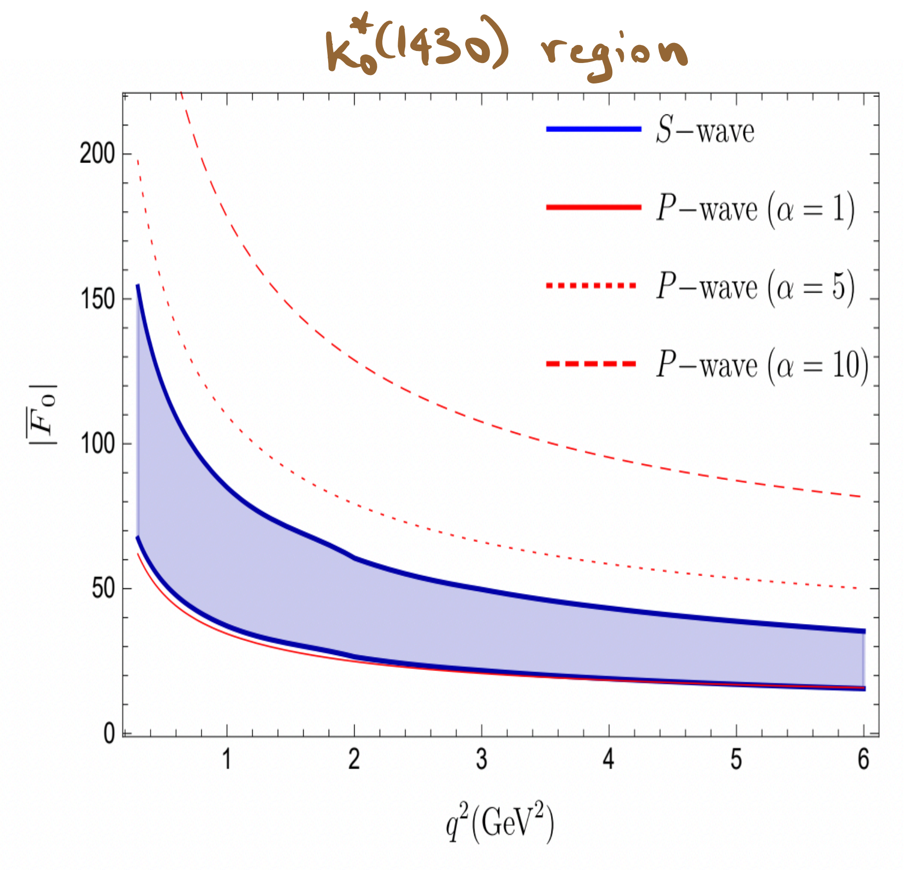
Relative size of S- and P-wave contributions





LCSRs for S-wave $B \to K\pi$ Form Factors

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S-wave fraction $F_S = BR(S\text{-wave})/BR(S + P\text{-wave})$